Eighth Grade Mathematics
Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the "how" and the "why" of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase "procedural skill and fluency" is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the "why" behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:

The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

The Number System (NS)

Standard 8.NS.A.1 Cluster Heading: A. Know that there are numbers that are not rational, and approximate them by rational numbers.
Know that numbers that are not rational are called irrational (e.g., π, √2, etc.). Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually or terminates and convert a decimal expansion which repeats eventually or terminates into a rational number.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Convert fractions to decimals. Convert terminating decimals to fractions. Differentiate between non-repeating, non-terminating decimals and repeating decimals. Differentiate between terminating and repeating decimals.	Convert repeating decimals to fractions. Identify a given number as rational or irrational.	Explain how irrational numbers differ from rational numbers. Determine when the decimal expansion of a fraction will terminate or repeat. Show that the decimal expansion of rational numbers eventually repeats or terminates.	Explain the relationship between numbers within the Real Number System using precise mathematical language.

Instructional Focus Statements

Level 3:

In grade 8, students perform operations with rational numbers to expand their understanding of the Real Number System by recognizing irrational numbers and their relationship to rational numbers. Students learn that numbers in the Real Number System are either rational or irrational and develop an understanding that when an irrational number is converted to a decimal, it is non-repeating and non-terminating. Likewise, when a rational number is converted to a decimal, it will repeat or terminate. To foster these understandings, students should access prior knowledge division from earlier grades to convert fractions to decimals. Instruction should also include opportunities recognize numbers such as square roots of non-perfect squares and pi into approximated decimals. Discussion should focus students to determine when the conversion results in a repeating or non-terminating decimal and students should be expected to use the proper notation for expressing these values. Additional practice converting fractions to decimals and discussion around the factors of their denominators will help as students eventually learn to predict when the decimal expansion of a fraction will be terminating. Just the same, students should engage in discussions around repeating patterns that occur when fractions have denominators of 3, 9, 99 or 11. Discussion should be facilitated so that students eventually discover that the decimal expansion of irrational numbers can only be approximated.

Instruction should build on what students already know about the hierarchy of the Real Number System. As students are introduced to irrational numbers and extend their understanding of rational numbers to include non-perfect squares and non-terminating decimals, a visual representation or diagram, such as a Venn diagram, will aid in their understanding of how rational and irrational numbers make up the Real Number System and how each subset of rational numbers relate to each other.

Students should also engage in discourse that leads to the understanding of why irrational numbers cannot be written as rational numbers. A common misconception is that all numbers in fractional form are rational. Therefore, students should be exposed to irrational numbers in fractional form such as $\frac{\pi}{2}$ and engage in discourse around fractional form and rational form.

Level 4:

Students at this level go beyond determining when a number is rational or irrational. They understand that real numbers are either rational or irrational and can explain why and how they make up the Real Number System. Students should be challenged to justify their explanations with illustrations and graphing organizers that include examples of rational and irrational numbers. In addition to discussing the relationship between rational and irrational numbers, students should be expected to explain the relationship between various types of rational numbers as well.

The Number System (NS)

Standard 8.NS.A.2 Cluster Heading: A. Know that there are numbers that are not rational, and approximate them by rational numbers.
Use rational approximations of irrational numbers to compare the size of irrational numbers locating them approximately on a number line diagram. Estimate the value of irrational expressions (such as π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Plot positive rational numbers on the number line. Order positive rational numbers using the number line. Compare positive rational numbers using the number line. Make comparative statements about the size of positive rational numbers. Evaluate numerical expressions containing positive rational numbers. Write the decimal equivalent of rational numbers.	Plot rational numbers on the number line. Compare rational numbers using the number line. Order rational numbers using the number line. Make comparative statements about the size of rational numbers. Evaluate numerical expressions containing rational numbers. Write the decimal approximation of irrational numbers.	Estimate the value of irrational numbers using rational approximations. Compare real numbers using the number line. Plot real numbers on the number line using their estimated values. Order real numbers using the number line. Make comparative statements about the size of irrational numbers. Estimate the value of irrational expressions.	Explain how to arrive at a more precise approximation when given an approximation of an irrational number.

Instructional Focus Statements

Level 3:

In grade 6 (6.NS.C.6) students located rational numbers on the number line and in grade 7 (7.NS.A.2d), students converted rational numbers to decimals using long division. In grade 8, students build on this learning to estimate values of irrational numbers and compare real numbers by positioning them on a number line. Using prior knowledge of the size of rational numbers, students should have opportunities to demonstrate finding rational approximations for irrational numbers such as $\sqrt{10}$. When estimating the value of an irrational number, discussion should include reasoning with the use of slightly larger or smaller rational numbers. In this case, they would explain that $\sqrt{10}$ is in between the two perfect squares $\sqrt{9}$ and $\sqrt{16}$, implying that $\sqrt{10}$ is between 3 and 4 and closer to, yet a little bit more than 3. Additional discussion should lead to the discovery of a more precise approximation as students estimate within which tenth the number lies, and then within which hundredth it lies.

Students should have multiple opportunities to plot real numbers on the number line and justify placement of the numbers using appropriate mathematical language. Students should also explain when a number's location should be approximated and justify comparisons of irrational numbers in relation to rational numbers. Placing irrational numbers on the number line will support students' understanding of standard 8.NS.A.1, where they learned that in addition to the rational numbers they previously worked with, irrational numbers are also part of the Real Number System. Students should be encouraged to refer to the number line as “the real number line” to emphasize the idea that there are an infinite number of real numbers represented by the line. Mathematical discourse should lead students to the understanding that between any two real numbers there are an infinite amount of numbers.

While students evaluated rational expressions in previous grades, they should now be exposed to expressions that contain irrational numbers such as 2π. Discussion should be facilitated in a manner that leads students to understand that the values of these expressions can only be estimated. Discussions should also include opportunities to construct viable arguments by justifying the estimated values of the irrational expressions. For example, knowing 3^2 is 9, students could argue that π^2 is slightly greater than 9. This new understanding of irrational numbers will be essential as students progress to grade 8 geometry and use π to find the area or circumference of a circle, or volume of a sphere. Students will also apply this understanding in grade 8 to solve problems involving the Pythagorean Theorem and non-perfect squares. Beyond grade 8, a thorough understanding of the Real Number System will support students' learning in high school as they begin to encounter solutions that are not part of the Real Number System.

Level 4:

Students at this level can not only estimate values of irrational expressions but can explain verbally and in writing how to improve a given estimation of the irrational expression. Given an irrational number, students can use a series of rational numbers to arrive at more and more precise estimations for the irrational number and can communicate this process using precise mathematical language. Students at this level should have opportunities to analyze an estimation of irrational numbers and discuss the process for extending the decimal to the hundredth, thousandth or further place to get a more precise estimation of the number.

Expressions and Equations (EE)

Standard 8.EE.A.1 **Cluster Heading: A. Work with radicals and integer exponents.**

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \, x \, 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
State that negative numbers raised to an even power can yield different products when parentheses are used. For example, $(-5)^2 = 25$ and $-5^2 = -25$. State that any number raised to the zeroth power has a value of 1. Write repeated multiplication of rational numbers using exponential notation.	Use the product rule with positive integer exponents to write equivalent numerical expressions using exponential notation. Use the quotient rule with positive integer exponents to write equivalent numerical expressions using exponential notation. Re-write a numerical expression, involving multiplication or division with positive integer exponents, as a simplified integer.	Use properties of integer exponents to generate equivalent numerical expressions (e.g., product rule, quotient rule, power rule, power of a product rule, zero exponent rule, and negative exponent rule). Rewrite numerical expressions with fractional bases raised to a power.	Explain why finding the power of a product results in each factor being raised to that power. Explain why any number to the zeroth power is 1 using known properties of integers and precise mathematical language. Explain how the properties of exponents apply to negative exponents.

Instructional Focus Statements

Level 3:

Students used whole number exponents to denote powers of 10 using place value in grade 5 (5.NBT.A.2). In grades 6 (6.EE.A.1) and 7 (7.EE.A.1) students wrote and evaluated numerical expressions with whole number exponents. They became familiar with exponential notation and learned that a number raised to a power (exponent) was a symbolic way to represent the repeated multiplication of that number. In grade 8, students continue to evaluate expressions with exponents, but now the exponents can be both positive and negative integers.

As students extend this thinking to develop an understanding of the properties of exponents, they should have opportunities to explore by expanding numerical expressions and observing patterns. Instruction should not focus on merely memorizing properties of exponents (or rules). Students should discover the properties through repeated reasoning. For example, expressions involving multiplication or division with like bases, $3^2 \cdot 3^5 = 3^7$. Multiple opportunities to practice expanding the factors $((3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3))$ will foster students' understanding of why we can add the exponents when multiplying with like bases and why the two expressions are equivalent. $(3^2 \cdot 3^5 = 3^{2+5} = 3^7)$. Students learn that a power of a power can be found by multiplying the exponents. For example, $(-6^2)^4 = (-6 \cdot -6) \cdot (-6 \cdot -6) \cdot (-6 \cdot -6) \cdot (-6 \cdot -6) = -6^8$. It is a common misconception for students to add these exponents, multiple opportunities to explore will aid in minimizing this misconception. This same practice is essential for understanding how each of the properties work. Though each property is unique, students should be challenged to apply multiple properties and re-write more complex expressions, for example, $\frac{(4^5)^{10} \cdot 4^{200}}{(4^2)^{20}}$. Furthermore, students should engage in discourse around the idea that there can be multiple expressions equivalent to the original one. Students will benefit from a solid understanding of these properties when they later explore scientific notation and make sense of very large and small numbers in standards 8.EE.A.3 and 8.EE.A.4.

Level 4:

At this level, students should be challenged to go beyond knowing and applying the properties of integer exponents. They should engage in discourse that prompts them to prove certain knowns related to integer exponents. For example, when finding the power of a product, students should be able to explain that each factor is raised to that power. Discussion should include their use of the associative and commutative properties to justify this using student generated examples.

Students know that a number raised to the zeroth power has a value of 1. Students should be probed to prove it using what they know about dividing exponential expressions with like bases. For example, they know that $\frac{5^2}{5^2} = 5^0 = 1$ can be written as 5^{2-2} or $5^0 = 1$. Although students' initial understanding of exponents includes positive integer exponents, additional practice should focus on explaining why these properties can also be applied in situations with negative exponents.

Equations and Expressions (EE)

Standard 8.EE.A.2 Cluster Heading: A. Work with radicals and integer exponents.

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize that a positive rational perfect square number is the product of two identical factors that can be written using a power of 2. Recognize that a positive rational perfect cube number is the product of three identical factors that can be written using a power of 3. Determine if a number is rational or irrational.	Identify the relationship between raising a number to second power (squaring a number) and taking the square root of a number as inverse operations. Identify the relationship between raising a number to the third power (cubing a number) and taking the cube root of a number as inverse operations. Know that the solution to the equation $x^2 = p$ can be a positive or negative value (x = $\pm\sqrt{p}$).	Identify the square root of a non-perfect square as irrational. Identify the cube root of a non-perfect cube as irrational. Evaluate square roots of small perfect square numbers. Evaluate cube roots of small, perfect cube numbers. Solve equations that require finding the square root of a number of the form, $x^2 = p$, where p is a positive rational small perfect square number. Solve equations that require finding the cube root of a number of the	Solve real-world problems that involve evaluating square roots and cube roots. Solve real-world problems that involve equations of the form $x^2 = p$ or $x^3 = p$, where p is a positive rational number. Create contextual problems that represent an equation of the form of $x^2 = p$ or $x^3 = p$, where p is a positive rational number. Explain why the square root of a non-perfect square or cube root of a non-perfect cube is irrational using precise mathematical language.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		form, $x^3 = p$, where p is a positive rational small perfect cube number.	

Instructional Focus Statements

Level 3:

Students used whole number exponents to denote powers of 10 using place value in grade 5 (5.NBT.A.2). They extended their understanding of whole-number exponents to evaluate numerical expressions in grades 6 (6.EE.A.1) and 7 (7.EE.A.1). In grade 8, students should be able to identify the relationship between raising a number to the second power (squaring a number) and taking the square root of a number as inverse operations and likewise, for raising a number to the third power and cube roots. It is essential for students to understand this relationship when they solve equations of the form $x^2 = p$ or $x^3 = p$, where p is a positive rational number. In solving these equations, students should be challenged to extend their understanding of the properties of equality and realize that taking the square root of one side of an equation requires taking the square root of the other side. As students solidify this understanding, they should be given opportunities to practice with positive rational numbers to discover that the same holds true when taking the cube root of one side of an equation.

As students develop a conceptual understanding of operating with square roots and cube roots, concrete visuals (e.g., square tiles and cubes) should be used to foster this understanding. Students should understand the relationship between perfect square numbers and the length of the side of a square and the same for perfect cube numbers and the length of the side of a cube. Discussion should focus on the relationship between repeated factors and expressing them as a base and an exponent. For example, the square root of 4 is 2 because the product of $2 * 2 = 4$, which can be expressed as $2^2 = 4$. These factors can also be connected to the side lengths of a square.

As students learn to evaluate square roots, they should understand that the solution to the equation $x^2 = p$ can be a positive or negative value ($x = \pm\sqrt{p}$). Discussion should be focused on the properties of multiplying negative numbers to help students determine that the evaluation of a cube root will yield one solution. In 8.NS.A.1, students learn that numbers that are not rational are called irrational numbers. Opportunities to evaluate square roots of perfect square numbers and square roots of non-perfect square numbers will support students' understanding of the relationship between rational numbers and irrational numbers. Additional practice with cube roots of perfect cube numbers and non-perfect cube numbers should be employed to solidify this understanding.

Level 4:

Once students have a strong understanding of square roots and cube roots, they should be expected to explain the process for solving equations with radicals using precise mathematical language. Instruction should challenge students to explain why the cube root of a non-perfect cube is irrational using precise mathematical vocabulary and supporting their arguments with known facts about rational numbers and/or models. Challenging students to connect what happens when a negative number is raised to a power to the effects on square and cube roots should help to push their understanding of possible positive and negative roots in either situation.

Students should be challenged to apply their understanding of operating with roots to solve real-world problems involving square roots or cube roots and provide arguments to support the practicality of their solutions. Additionally, they should be provided opportunities to create real-world problems that can be solved by writing an equation in the form of $x^2 = p$ or $x^3 = p$. For example, "A square garden has an area of 225 sq. ft. What is the length of each side?"

Equations and Expressions (EE)

Standard 8.EE.A.3 **Cluster Heading: A. Work with radicals and integer exponents.**

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a number expressed in scientific notation to represent a small or large number. Estimate powers of 10.	Estimate very small or very large quantities using numbers that are represented in scientific notation. Translate between numbers written in standard form and scientific notation.	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities. Indicate how many times larger one number represented in scientific notation is than a second number also expressed in scientific notation.	Solve real-world and mathematical problems by comparing estimated quantities represented in scientific notation and explain the benefits for using scientific notation using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

As students develop an understanding of the properties of exponents (8.EE.A.1), they should extend their understanding to be able to write very large and very small numbers in scientific notation using positive and negative exponents. Scientific notation uses exponents to write very large or very small numbers as a product of two factors, the first factor being a single quantity that is greater than or equation to 1 but less than 10 and the second factor being an integer power of 10. For example, 200,000 written in scientific notation is 2×10^5 and 0.0007 written in scientific notation is 7×10^{-4}. Students should develop a conceptual understanding of how and why a number is written in scientific notation. The exponent tells the greatest place value of the number, not the number of zeros in the standard form of the number. Another common misconception is for students to confuse a very large number for a very small number when written in scientific notation. Rather than rules, draw on students' background knowledge of negative exponents by asking students to look at the exponent first to determine whether it is a small or large number. Additionally, students should understand the connection between a number written in scientific notation and the properties of exponents rather than memorizing a set of procedural rules.

Introduce students to examples of very large and very small numbers in contexts. Seeing these values in real-world situations will help students to compare numbers written in scientific notation to determine how many times larger (or smaller) one number is than another. When comparing, students should understand that if the exponent increases by 1, the value increases 10 times. Referencing the population example given in the standard, students can compare 10^8 to 10^9 by understanding that exponent increased by 1. Also, compare the first factor of each product to see that 7 is a little more than 2 times 3. Calculating 2 x 10 demonstrates that the world population is about 20 times larger than the United States population.

Level 4:

Students should extend their understanding of the properties of exponents by solving real-world and mathematical problems involving numbers written in scientific notation. Students should also be able to compare numbers involving scientific notation and explain the comparison between the numbers by determining how many times larger or smaller one is than the other. Additionally, students should understand and explain the benefits of using scientific notation. This standard lays the foundational coursework for the sequential standard (8.EE.B.4) for which students will perform operations with numbers expressed in scientific notation.

Equations and Expressions (EE)

Standard 8.EE.A.4   Cluster Heading: A. Work with radicals and integer exponents.

Using technology, solve real-world problems with numbers expressed in decimal and scientific notation. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities *(e.g., use millimeters per year for seafloor spreading)*.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an expression in scientific notation that represents a decimal number and vice versa. Estimate very small or very large quantities using numbers that are represented in scientific notation.	Perform operations with numbers expressed exclusively in scientific notation using technology. Express a number written in decimal form in scientific notation and vice versa.	Choose units of appropriate size expressed in scientific notation to represent measurements of very large or very small quantities in a real-world context. Using technology, solve real-world one-step problems that involve performing operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used.	Generate units of appropriate size expressed in scientific notation to represent measurements of very large or very small quantities. Using technology, solve real-world multi-step problems that involve performing operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Interpret scientific notation that has been generated by technology.

Instructional Focus Statements

Level 3:

Students should extend previous knowledge of the properties of exponents (8.EE.A.1), and of writing very large and very small numbers in scientific notation (8.EE.A.3) to solve problems that involve performing operations with numbers in scientific notation. Students should realize that quantities in real-world problems can be expressed in scientific notation or decimal form and be used interchangeably within real-world problems. This standard calls for students to use technology to be able to compute operations with numbers in both decimal and scientific notation and interpret generated values. When solutions are generated by technology, discussion should continue to build on previous knowledge of very large and very small numbers in scientific notation (8.EE.A.3) as well as the context of the real-world problem to assess the reasonableness of answers.

Students should also focus on the size of the measurement to choose which units are appropriate for the contextual situation. When working with a real-world situation involving distances, students attend to precision (MP6) by communicating using scientific and/or decimal notation and accurate unit labeling. Students might need to determine the unit of measure that would be most accurate and appropriate to measure distances. For example, to measure the length of one paper clip, students might choose centimeters as an appropriate measurement. To determine how many paperclips it would take to make a line from New York City to Disney World, kilometers may be a more appropriate unit of measure.

Level 4:

Students should extend their understanding of performing operations with numbers in scientific notation to solving multi-step contextual problems that involve both numbers in scientific notation form and decimal form. Additionally, students should be able to perform calculations using technology that produce a solution in scientific notation and be able to interpret the solution and explain their reasoning using precise mathematical vocabulary.

Equations and Expressions (EE)

Standard 8.EE.B.5 **Cluster Heading: B. Understand the connections between proportional relationships, lines and linear equations.**
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a graph that represents a given proportional relationship.	Identify the unit rate as the slope in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	Graph a given proportional relationship. Identify the slope from a provided graph of a proportional relationship and connect it to the unit rate. Compare two different proportional relationships represented in different ways.	Compare two proportional relationships generated from different contexts in terms of the contexts they represent. Identify the context of the unit rate from the graph of a proportional relationship generated from a contextual problem.

Instructional Focus Statements

Level 3:

Students build on prior conceptual understanding of unit rates in grade 6 (6.RP.A.2) and proportional relationships in grade 7 (7.RP.A.2) to compare graphs, tables, and equations of proportional relationships. Students should develop a solid foundation in graphing proportional relationships and determining and interpreting unit rate and constant of proportionality as the slope of a graph. This standard calls for students to compare two different proportional relationships represented in different ways. For example, compare the unit rate in a contextual problem about a cell phone bill represented graphically on a coordinate plane to a cell phone bill from a different company where the unit rate can be found represented in a table. Students can make use of structure (MP7) of a representation to compare different representations by perhaps displaying both sets of information using the same representation.

As students work with different representations when comparing the unit rates as the slope of the proportional relationships, it may be an overwhelming amount of information to analyze. Scaffolds, purposeful questioning, and pressing students to justify each slope in the proportional relationships can be used to ensure that students have a strong conceptual understanding of each relationship being compared. Use questions such as, *"What is happening in each situation? How are they the same? Different? What do the slopes tell us about the situation?"* When making comparisons, students should be flexible in using different strategies and not memorize a set of steps as a comparison process.

Level 4:

Students should employ their knowledge of unit rates and proportional relationships to compare two different proportional relationships in complex situations. Students should be able to write and verbalize their explanation of unit rate as the slope of a graph in a complex problem. Additionally, students should display procedural fluency by understanding that when provided information presented in a table and information presented in equation form, the two may be easier compared by graphing both relationships. As students solidify this procedural fluency, students should be able to justify their reasoning with written and verbal explanation.

Equations and Expressions (EE)

Standard 8.EE.B.6 Cluster Heading: B. Understand the connections between proportional relationships, lines and linear equations.

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; know and apply the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the y-intercept of a given graph.	Choose an equation in the form $y = mx + b$ or $y = mx$ to represent a line graphed on a coordinate plane. Find the slope of a line, given two points on the line.	Give an equation in the form $y = mx + b$ or $y = mx$ to represent a line graphed on a coordinate plane. Explain that the slope is the same between any two points on a line using similar triangles. Choose a representation demonstrating that the slope is the same between any two points on a line using similar triangles.	Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Instructional Focus Statements

Level 3:

Students should employ prior knowledge of proportions and slope (8.EE.B.5) to develop a conceptual understanding of why the slope of a line remains constant. Students develop this understanding by working with similar triangles that are formed by the vertical and horizontal lines from a point on a non-vertical line. Students should be exposed to multiple representations of this concept as they develop an understanding that similar “slope” triangles have equivalent side length ratios. Additionally, they should develop an understanding that the slope is the same between any two points on a line using similar triangles. As students work with this concept, they should gain an understanding of the equation of the line, $y = mx + b$, where m is the slope and b is the y-intercept. Students should be able to identify the slope of a line from graphs, tables, and equations and make connections between the representations.

Students should move beyond finding the slope of a line. They should use multiple representations to demonstrate and explain why any two points on a non-vertical line generate the same slope by using similar slope triangles. Additionally, students should be able to graph the equation of a line, identify the slope, and create a visual representation that shows that the slope is the same between any two points on a line using similar triangles formed by the vertical and horizontal lines from a point on a non-vertical line.

Level 4:

As students solidify their understanding of this concept, they should be able to derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b by making use of structure from tables, equations, and graphs.

Equations and Expressions (EE)

Standard 8.EE.C.7 **Cluster Heading: C. Analyze and solve linear equations, linear inequalities, and systems of two linear equations.**

Solve linear equations in one variable.

8.EE.C.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve linear equations in the form $x + q = r$ or $px = r$.	Determine if a given linear equation in one variable has no solution, one solution, or infinitely many solutions. Solve linear equations in the form $px + q = r$ or $p(x + q) = r$.	Give examples of linear equations in one variable having one solution, infinitely many solutions, or no solution. Solve linear equations with rational coefficients whose solutions require expanding expressions using the distributive property and collecting like terms.	Explain why a linear equation has no or infinitely many solutions. Solve linear equations with rational coefficients whose solutions require expanding expressions using the distributive property and collecting like terms and explain the operations used in each approach.

Instructional Focus Statements

Level 3:

Students should use prior knowledge of equality properties and equivalence for solving equations (6.EE.B.7 and 7.EE.B.4a) to solve more complex linear equations in one variable with coefficients that include integers, fractions, and decimals can be solved by expanding expressions with the distributive property and/or combining like terms and equations with variables on both sides. Students should also determine if a linear equation results in one, zero, or infinitely many solutions. To visualize solving equations, encourage students to demonstrate equations using concrete objects such as algebra tiles or balance scales, as well as with visual representations including drawing pictures, diagrams, bar models, etc. It is imperative that students develop conceptual understanding of the various solution types building on the fact that solutions to equations maintain equality and how the solutions are related to the linear equation through a discovery process as opposed to a rote set of memorized rules.

Students should begin to develop a conceptual understanding of the implications when the solution for a linear equation results in one, zero, or infinitely many solutions. Students should be able to give examples of the different solution types to linear equations. Additionally, these standards lay the foundation for future coursework with linear equations and set a precedent for understanding with the other function types students will experience in later courses. Thus, it is imperative for students to gain an in-depth conceptual understanding of the inner workings of linear equations.

Level 4:

Students should extend their understanding of solving linear equations utilizing a variety of multiple properties of operations by explaining their solution approach using precise mathematical vocabulary in verbal and written forms.

Students should solidify their conceptual understanding of what it means when the solution for a linear equation results in one, zero, or infinitely many solutions. Students should understand that when $x = a$, there is only one solution and that substituting the value of a into the equation will result in a true equation. Students should also be able to understand that when $a = a$, there are infinitely many solutions and substituting any number into the equation will result in a true equation. In the same manner, when $a = b$ (where a and b are different numbers), there are no solutions, and any number substituted into the equation will result in a false equation.

The culmination of these standards will be the building block for future work solving pairs of simultaneous linear equations.

Equations and Expressions (EE)

Standard 8.EE.C.8 **Cluster Heading: C. Analyze and solve linear equations, linear inequalities, and systems of two linear equations.**

Analyze and solve systems of two linear equations graphically.

8.EE.C.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.C.8b Estimate solutions by graphing a system of two linear equations in two variables. Identify solutions by inspecting graphs.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Graph linear equations in two variables on the coordinate plane. Identify points on a line as solutions to their corresponding equation. Graph a line on the coordinate plane when given a pair of coordinates.	Determine if a linear system of equations has one solution, infinitely many solutions, or no solution when given a graph. Determine the slope and y-intercept when given a linear equation.	Analyze a system of linear equations to determine if there is one solution, no solution or infinitely many solutions. Estimate solutions to a system of equations by graphing. Identify solutions to a system of linear equations by inspecting graphs. Explain how the number of solutions of a system relates to the context of a real-world problem.	Explain characteristics of systems that have one solution, no solution or infinitely many solutions using precise mathematical vocabulary. Justify the reasonableness of the solution to a system of equation in context by inspecting graphs.

Instructional Focus Statements

Level 3:

In grade 7, students used linear equations in one-variable to solve real-world and mathematical problems (7.EE.B.4). Students also defined the meaning of the variable and explained the solution in terms of the given context. In grade 8, students should build on that knowledge to solve linear equations in one variable and determine if the equation will have one solution, infinitely many solutions, or no solutions (8.EE.C.7). This prior knowledge can be applied to help achieve mastery of this standard which requires students to begin exploring systems of equations, where they graphically solve two equations simultaneously, understanding that a system of linear equations can also have one, many, or no solutions. Instruction in grade 8 will focus on exploring systems of equations graphically to develop a conceptual understanding of the solution(s). Students should have opportunities to make connections between systems with no solution, one solution, and infinitely many solutions and how those relate to the context of a real-world problem. Solving systems algebraically is not an expectation of grade 8.

Using graphs, students should conceptually understand that the intersection of two lines on a graph represent the solution to the system of linear equations and be able to explain why this is true. Discussions should include careful inspection of the graph to allow students to see that the solution, or point of intersection, is an ordered pair that lies on both lines. Students should be exposed to system of linear equations with lines that intersect, but also with lines that are parallel. Facilitate discussion about what the graph would look like if there were no solutions (parallel lines that never intersect) and if there are an infinite number of solutions.

Students know that the slope of a line describes its rate of change (8.EE.B.5) and the slope is the same between any two distinct points on a non-vertical line (8.EE.B.6). Applying this knowledge, students should be pressed to describe the lines represented by pairs of coordinates and/or write the equations of the lines that include these points. Inspection of coordinates and equations will support students' ability to reason about the number of solutions as they should determine that lines with different slopes will have one solution, lines with the same slope and different y-intercepts have no solutions, and lines that have the same slope and same y-intercept will have infinitely many solutions.

A strong understanding of systems of linear equations and their solutions will support students as they continue this learning in high school and go on to write and solve systems of linear equations in a real-world context (A1.A.REI.C.4 and M1.A.REI.C.3). Students in high school courses will build on this conceptual understanding of graphical solutions to systems of equations from grade 8 by solving systems algebraically, such as using substitution and elimination methods. These algebraic solution methods will not be assessed in grade 8.

Level 4:

At this level, students with a strong, conceptual understanding of systems of linear equations should be able to make connections about the number of solutions a system would have by examining the graph and explain using precise mathematical language. Anytime students at this level identify a system of equations has no solution, they should be able to explain, using precise mathematical language, the reason for this and justify their answer graphically. Students with a deep level of understanding can graphically solve systems of equations created from a context and justify their solutions by connecting the solution back to the text. Discussion could focus on having students not only justify the solution mathematically but recognize the reasonableness of the solution in terms of the context.

TN Department of Education

Equations and Expressions (EE)

Standard 8.EE.C.9 **Cluster Heading: C. Analyze and solve linear equations, linear inequalities, and systems of two linear equations.**

By graphing on the coordinate plane or by analyzing a given graph, determine the solution set of a linear inequality in one or two variables.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify if a point on a number line is a solution to a linear inequality in one variable. Identify if a point on a coordinate plane is a solution to a linear inequality in two variables.	Choose a number line graph that represents the solution set for an inequality in one variable. Choose the graphical representation of the solution to a linear inequality in one variable. Choose the graphical representation of the solution to a linear inequality in two variables.	Determine the solution set to a linear inequality in one variable by graphing on a number line. Determine the solution set to a linear inequality in one variable by graphing on the coordinate plane. Determine the solution set to a linear inequality in two variables by graphing on the coordinate plane. Analyze a given graph to determine linear inequality solution sets.	Justify whether coordinate points are or are not solutions to the linear inequality in two variables using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6 (6.EE.B.8), students learned to interpret and write simple inequalities, as well as graph solutions of simple inequalities on number lines. In grade 7 (7.EE.B.4b), students learned to solve two-step inequalities in one variable and graph the solution set on a number line. In grade 8, students will use this prior knowledge to be able to graph the solution set of a linear inequality on the coordinate plane. Instruction should focus on extending a student's understanding of graphing linear inequalities in one variable on a number line and graphing linear equations in two variables on a coordinate plane, to graphing linear inequalities in one or two variables on a coordinate plane.

Students should understand that the solutions to a linear inequality can be represented by many points on a coordinate plane and that the set of points are in a region bounded by a line. To represent the set of all points that are solutions, the region is shaded. Students should also understand that the points that fall on the non-shaded side of the boundary line are not solutions to the inequality. Similar to using an open circle for $>$ and $<$ and closed circle for $\leq$ and $\geq$ on a number line, the boundary line on a coordinate plane is dashed for $>$ and $<$ and solid for $\leq$ and $\geq$. A solid line means that all coordinate points on the line are solutions, and a dashed line indicates points on the line are not included as solutions. It is important for students to test points on either side of the line to see if the pair of values make the inequality true, or to reason carefully about the inequality statement and think about pairs of values that would satisfy the inequality.

Students need to understand that graphing a one-variable inequality can result in a horizontal or vertical line.

Level 4:

Students should be able to justify whether coordinate points are or are not solutions to the linear inequality in two variables using precise mathematical vocabulary. It is important for students to test points on either side of the line to see if the pair of values make the inequality true, or to reason carefully about the inequality statement and think about pairs of values that would satisfy the inequality. This standard builds a foundation of understanding for students to graph the solution set to a system of linear inequalities in two variables in high school (A1.A.REI.D.7 or M1.A.REI.D.6).

TN Department of Education

Functions (F)

Standard 8.F.A.1 **Cluster Heading: A. Define, evaluate, and compare functions.**

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in 8th grade.)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine that a relation is a function or not a function given a set of ordered pairs or a table.	Explain that a function is a rule that assigns to each input exactly one output and justify their thinking using either a set of ordered pairs, a table of values, or a graph. Determine that a relation is a function or not a function given a graph.	Explain that a function is a rule that assigns to each input exactly one output and justify their thinking using a set of ordered pairs, a table of values, and a graph. Determine that a relation is a function or not a function.	Explain why a relation sometimes is and sometimes is not a function. Create a function or non-functional relationship and provide justification. Make connections between functions and non-functional relationships.

Instructional Focus Statements

Level 3:

In this standard, students are introduced to functions as rules that assign exactly one output to each input. This is the first formal introduction of functions to students. Informally, in previous grade levels, students develop foundational understandings by generating patterns that follow a given rule. In grades 6 and 7, students work with ratios and proportional relationships using tables, equations, and graphs. In grade 8, students should develop a firm foundational understanding that functions describe situations in which one quantity is determined by another. When describing the relationship between input and output quantities, students develop an understanding that the defining characteristic of a function is that the input value determines the output value, or vice versa, that the output depends upon the input value. Students should be able to reason whether a relation presented as a table, graph, or set of ordered pairs models a function or not. To contextualize the concept of a function, it is often beneficial to relate a function to a tool that allows a number to be put in and only allows one number to be put out. It is important that students be allowed to develop a conceptual understanding of the concept of which relations can be defined as functions as opposed to being presented with a set of rules applicable to a particular relation representation. For example, the vertical line test should result from discovery learning as opposed to being presented as a rule to determine if a graph is a function or not.

Level 4:

As students extend their foundational understanding of functions, they should begin to make generalizations about ordered pairs represented in tables, graphs, and equations. For example, students should discover through repeated reasoning (MP 8), that a relation represented in a graph where two or more *x* coordinates are the same for any of the points is never a function since this would contradict the definition of function. These generalizations should be made with verbal and written form justifications and supported with visual representations.

Functions (F)

Standard 8.F.A.2 **Cluster Heading: A. Define, evaluate, and compare functions.**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and another linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine which function has the greater rate of change, given two linear functions both represented graphically. Determine the intercepts of each function given tables and graphs.	Compare properties (rate of change and intercepts) of two functions, each represented in the same way algebraically, graphically or numerically in tables. Determine which function has the greater rate of change, given two linear functions both represented algebraically in slope-intercept form.	Compare properties (rate of change and intercepts) of two functions, each represented in different ways algebraically, graphically, numerically in tables, or by verbal descriptions.	Compare properties of two functions, each represented in different ways, when the functions are embedded in a contextual problem. Compare properties of multiple functions, each represented in different ways algebraically, graphically, numerically in tables, or by verbal descriptions, and flexibly move between representations to determine key features. Given a contextual situation, write a function and interpret the rate of change and intercepts in terms of the context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			Create contextual situations that involve comparing the properties of two functions each represented in different ways and use precise mathematical language to describe the comparisons.

Instructional Focus Statements

Level 3:

Previously, in grade 7, students recognized an important type of regularity in numerical tables by discovering the multiplicative relationship existing between each pair of values in tables generated from equations in the form y = cx and further identifying the constant of proportionality. As students enhance their understanding of functions in grade 8, they should be able to compare properties represented in the same and different forms using tables, graphs, equations, and contextual situations. To further develop the idea of rate of change (slope), students should be able to compare the rate of change of two functions in multiple representations. These comparisons should be explained and supported with verbal and written explanation as well as visual representation is different forms. It is also important to note that there are other properties that can be compared beyond the rate of change. Though an important relationship that exists within functions, rate of change should not be the exclusive focus for this standard. Students should compare the *x*- and *y*-intercepts when functions are given as tables, graphs and as equations. Mathematical discourse should lead students to discover the relationship between functions and their intercepts, both given a context and given in various mathematical representations.

Level 4:

Students should solidify their understanding of functions and comparisons of properties in functions in contextual problems. Students should be able to deconstruct a contextual situation that involves comparing the properties of two functions and represented each in different ways, providing a justification on why they have represented the function in a certain way to elicit certain properties. Students at this level should be able to move flexibly between representations. This should be done using precise mathematical language to describe the comparisons.

Functions (F)

Standard 8.F.A.3 **Cluster Heading: A. Define, evaluate, and compare functions.**

Know and interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a function is linear when a graph is provided. Determine if a function is non-linear when a graph is provided.	Determine if a function is linear when an equation is provided. Determine if a function is non-linear when an equation is provided.	Distinguish between a linear function in the form $y = mx + b$ and a non-linear function. Provide examples of linear and non-linear functions.	Explain why a function is linear or non-linear. Explain the similarities and differences between linear and non-linear functions in both verbal and written form providing examples of both to justify their thinking. Re-write the equation in slope – intercept form and identify that it is a linear function and subsequently graph as a straight line when given a linear equation not in slope-intercept form.

Instructional Focus Statements

Level 3:

The focus of this standard is for students to become familiar with the slope-intercept form of a linear equation ($y = mx + b$) as defining a linear function that will graph as a straight line. Through repeated reasoning (MP 8), students should discover similarities and differences of linear and non-linear functions in both algebraic and graphical representations. Students should be given the opportunity through extended discussions to compare the differences in linear and non-linear functions and relate those differences back to the equation and the graph of the two functions. Students in grade 8 should also be able to flexibly interpret the slope and y-intercept values including functions containing integers, fractions and mixed numbers Linear functions are a major focus of this standard, but students are also expected to give examples of functions that are not linear.

A common misconception for students is to misclassify a function as nonlinear when the y-intercept is zero (e.g., $y = 3x$). Student should have opportunities to create a table of values and graph the values to confirm linear equations in the form $y = mx + 0$ are straight lines and are linear functions. Teachers may choose to emphasize the connection between slope-intercept form of an equation and a b value of zero by expressing the function as (e.g., $y = 3x + 0$).

Level 4:

As students discover similarities and differences of linear and non-linear functions, they should make generalizations about each and explain their thinking using precise mathematical language accompanied by visual representations to support their reasoning. Additionally, as the standard indicates students should know the equation $y = mx + b$ as defining a linear function, students should be able to make the connection that x and y are the input and output values, m is the multiplicative relationship, and b is the constant or initial value. It is also important for future course work with parent functions that students understand that the "b" (initial value) is a constant value that indicates the number of units the linear function shifts vertically.

Functions (F)

Standard 8.F.B.4 **Cluster Heading: B. Use functions to model relationships between quantities.**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the rate of change and initial value when given the equation of a linear function in the form $y = mx + b$. Choose a function that models a linear relationship when provided a graph.	Interpret the rate of change and initial value in terms of a situation when given the equation of a linear function in the form $y = mx + b$. Determine the rate of change of a linear function when given a graph. Determine the rate of change of a linear function when given two (x, y) values. Determine the initial value of a linear function from a table or graph that contains the initial value.	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of a linear function when given a table. Determine the rate of change and initial value of a linear function when given a graph. Determine the rate of change and initial value of a linear function when given two (x, y) values. Interpret the rate of change and initial value of a linear function in terms of the situation it models.	Create a function to model a linear relationship representing a contextual situation and interpret the rate of change and initial value in terms of the situation it models. Make connections about the rate of change when a function is represented in a table, graph, algebraic form, or by verbal descriptions. Create a real-world problem which can be modeled by a linear relationship whose solution requires either an interpretation of the rate of change or initial value in terms of the situation it models.

Instructional Focus Statements

Level 3:

The instruction of this standard should be focused around increasing students' conceptual understanding of linear functions by building on the knowledge students have from working with the other function standards in grade 8. The end goal is for students to extend their understanding of linear functions to work with linear functions embedded in contextual situations. Students should be able to construct linear functions from information presented in a wide variety of ways. Crucial to this is a student's ability to calculate the slope of a line from information presented in a wide variety of ways and to identify the initial value regardless of how information is presented. Both are imperative as students progress to writing the linear function represented. Students should also discover the connections that exist between the graphed *y*-intercept and the initial value of the function. Students should be able to find the initial value from a table where there is no $x=0$ value given. Ultimately, students should be able to write linear functions whose information is embedded in a contextual situation and then explain the meaning of slope and the *y*-intercept in contextual problems using precise mathematical vocabulary.

Level 4:

As students deepen their understanding of constructing functions, they should also be able to create a function to model a linear relationship representing a contextual situation and interpret the rate of change and initial value in terms of the situation it models. As students extend their learning, they should be able to make connections about the rate of change and initial value when a function is represented in different forms, including in a table, graph, equation, or by verbal description.

Functions (F)

Standard 8.F.B.5 **Cluster Heading: B. Use functions to model relationships between quantities.**

Describe qualitatively the functional relationship between two quantities by analyzing a graph (*e.g., where the function is increasing or decreasing, linear or nonlinear*). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a functional relationship is linear or non-linear, given a graph.	Determine if a linear function is increasing or decreasing, given a graph.	Qualitatively describe the functional relationship existing between two quantities when given a linear or non-linear graph. Sketch a graph that represents a function given a written or verbal description, and label the axes appropriately.	Create contextual situations that models a relationship between two quantities and describe qualitatively the functional relationship between the two quantities. Sketch the graph, labeling the axes appropriately. Determine the functional relationships that exist for each piece, given a piecewise graph (i.e., which portions of the graph represent a linear relationship, which represent a non-linear relationship, which are increasing, and/or which are decreasing).

TN Department of Education

Instructional Focus Statements

Level 3:

The instructional focus of this standard should be helping students understand the connection that exists between a written or verbal description of important traits of a function and the graph of the function. This should include where the function is increasing or decreasing, and if the function is linear or nonlinear. Students should also be able to label and interpret the axes with respect to the provided verbal description. Similarly, students should be able to sketch a graph that exhibits the qualitative features of a function given a written or verbal description of the function. As students develop a conceptual understanding of relationships between quantities and describing the relationships qualitatively, students should pay close attention to the shape of the graph rather than the specific numerical values and analyze the qualitative attributes from left to right describing what happens to the output as the input increases. Students should have opportunities to explore features of a graph of a function within specific intervals. While not an expectation of this standard, teachers might consider introducing students to the notation $0<x<2$ as a more concise way to write x is between 0 and 2. This notation will be formally introduced in future coursework. A common example is a verbal description or graph of a plane that travels from one point to another where the y-axis indicates the height of the plane and the x-axis indicates time. Students should use precise mathematical language when describing these relationships.

Level 4:

To enhance understanding, students should be able to create their own contextual situations that model a relationship between two quantities. They should describe qualitatively the functional relationship between the two quantities, attending to precision (MP 4), using mathematical vocabulary. As students sketch graphs to model the contextual situation, they should accurately be able to label the axes and explain what each means with respect to the context. To strengthen understanding, students should be able to assign specific numerical values to the graph, representing a contextual situation, and provide an explanation of the range indicating where the graph is specifically increasing and decreasing.

Students may also be given opportunities to extend their understanding of functional relationships given piecewise functions. Analyzing the pieces of the piecewise function (linear or nonlinear, increasing or decreasing) and how those pieces describe a given situation may help deepen their understanding of functions. Students may also be given a graph of a piecewise function and be asked to create a situation that may be modeled using the given graph. They may also be given a situation that can be described using a piecewise function and determine what the graph may look like.

Geometry (G)

Standard 8.G.A.1 Cluster Heading: A. Understand and describe the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles.

Describe the effect of translations, rotations, reflections, and dilations on two-dimensional figures using coordinates.

8.G.A.1a. Verify informally that lines are taken to lines and determine when line segments are taken to line segments of the same length.

8.G.A.1b. Verify informally that angles are taken to angles of the same measure.

8.G.A.1c. Verify informally that parallel lines are taken to parallel lines.

8.G.A.1d. Make connections between dilations and scale factors.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Generate figures using coordinates on the coordinate plane. Accurately measure angles and line segments. Informally define translation, rotation, reflection, and dilation. Identify when figures are congruent, similar, or neither.	Identify if an image and a pre-image represent a translation, rotation, reflection, or dilation. Identify which points, lines, and angles in the pre-image map onto which points, lines, and angles in the image after a transformation. Translate, rotate, reflect, and dilate a line segment on a coordinate plane. Identify when transformations maintain congruence or result in a similarity.	Transform a two-dimensional figure on the coordinate plane using translations, rotations, reflections, and dilations. Describe the effect of translations, rotations, reflections, and dilations on two-dimensional figures using coordinates. Informally verify the transformation used when mapping one figure onto another on the coordinate plane. Informally verify that line segments remain congruent after translations, rotations, and reflections, but	Verify and explain the effects of rigid and non-rigid transformations using coordinates and precise mathematical language. Compare and contrast the transformations: rotation, reflection, translation, and dilation using precise mathematical vocabulary.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		change length after dilations. Informally verify that angle measures remain the same after translations, rotations, reflections, and dilations. Informally verify that parallel lines remain parallel after translations, rotations, reflections, and dilations. Describe the effect a dilation will have on an image and its coordinates when given the scale factor. Use the correct notation when labeling the pre-image and image and when describing a transformed figure.	

Instructional Focus Statements

Level 3:

In grade 7, students learned about scale drawings in mathematical and real-world problems (7.G.A.1). In grade 8, this prior learning can be connected to the concept of dilations. Like scale drawings, students need to recognize that dilations maintain the same shape (similar figures) as opposed to the figure getting stretched or compressed. A quick exploration of scaling clip art or a digital photograph can help students understand these differences. This comparison will also help prepare students for applying stretches, compressions, and translations to functions in high school (A1.F.BF.B.2/M2.F.BF.B.2 and A2.F.BF.B.3/M3.F.BF.A.2). Instruction should limit dilations to those with the center at the origin in grade 8, so the focus can be a simple introduction with a chance to explore. Students will build on this introduction to dilations in high school (G.SRT.A.1/M2.SRT.A.1).

In this standard, students will also explore three other transformations: rotations, reflections, and translations. Students should spend time exploring and comparing each of these transformations using tools that help them understand the action taking place, such as patty paper, tracing paper, transparencies, graph paper, manipulatives, mirrors, compasses, protractors, rulers, and technology. For example, students should informally recognize that in a translation, the line segments of the figure in the image will always be either parallel to or lie on the same line as the corresponding lines of the figure in the pre-image. This will not always be true after a rotation or reflection.

Discussion should focus on what students notice about the new figure in comparison to the original. Students should engage in discourse around the characteristics of the figures (i.e., angle measures, parallel lines, lengths of line segments) both before the transformation (pre-image) and after (image). For example, if parallel lines exist in the pre-image, they will still be parallel in the resulting image with any of these transformation types. However, they may or may not maintain their lengths. This should lead to an understanding of which of these transformations produce images congruent to the pre-image and which produce similar figures. Use of correct mathematical vocabulary and notation, such as *A* for the pre-image and *A' (A' read as "A prime")* for the image, as well as the parallel symbol || and congruence symbol ≅, is expected.

As students' understanding of each of these transformations is solidified, they should be challenged to identify which transformation was used when mapping a pre-image to an image. Student discourse should be focused on which line segments and angles in the pre-image map to which lines and angles in the image and whether they maintain congruency. Students should also be challenged to verify that parallel lines map to parallel lines by comparing slopes.

While introduction to these transformations should occur off a coordinate plane, instruction should eventually guide students to draw the transformations in the coordinate plane, using the coordinates to track the figure before and after the transformation and provide a way to show and communicate the effects. This informal exploration will build a foundation for students on which they will develop a formal definition of each transformation in high school (G.CO.A.3/M2.G.CO.A,3). In grade 8, students should only work with single transformations. Reflections in the coordinate plane are limited to the axes as the line of reflection. Rotations are limited to 90-degree intervals both clockwise and counterclockwise. They will expand to a more formal exploration of transformations, including a sequence of transformations, in high school (G.CO.A.4/M2.G.CO.A.4).

Level 4:

Students at this level should use the coordinates to verify and explain the effects of each type of transformation using precise mathematical language, including how they know which are rigid transformations (those that result in congruent figures) and which are non-rigid transformations (those that do not result in congruent figures). They can also use the coordinates to compare and contrast the effects of each type of transformation.

TN Department of Education

Geometry (G)

Standard 8.G.A.2 Cluster Heading: A. Understand and describe the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles.

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize vertical, adjacent, and supplementary angles. Identify the exterior angles of a triangle. Find the measure of a missing angle in a pair of vertical angles or adjacent supplementary angles when given one angle measure. Identify parallel lines and the intersection of a transversal.	Know that the sum of the angles in a triangle is always 180 degrees. Identify the relationship between an interior and an exterior angle of a triangle as supplementary. Use facts about angle relationships to find a missing interior angle measure of a triangle. Define similar figures.	Informally explain the triangle sum theorem using a strategy. Give informal arguments to establish facts about exterior angles of triangles. Find and justify missing interior and exterior angle measures of a triangle using facts about angle relationships. Informally explain the relationship of angles created by parallel lines cut by a transversal. Identify pairs of congruent angles and pairs of supplementary angles formed by parallel lines cut by a transversal.	Justify mathematically the relationships of lines and angles created by parallel lines cut by a transversal. Create and solve a real-world contextual problem involving triangles. Create and solve a real-world contextual problem involving angles formed by a pair of parallel lines cut by a transversal.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Use informal arguments to establish facts about the angle-angle criterion for similarity of triangles. Recognize that if two triangles have two pairs of corresponding congruent angles, then they are similar.	

Instructional Focus Statements

Level 3:

In grade 8, students build on their experience from grade 7 with triangles (7.G.A.2) and pairs of angles (7.G.B.4) to construct informal arguments about their properties while exploring facts about the angles of a triangle and angles created when parallel lines are cut by a transversal. Students will also extend that understanding to discover the angle-angle criterion for similar triangles.

Given the opportunity to build triangles using manipulatives and examine the interior angle measures of a triangle will solidify students' understanding of the triangle sum theorem leading them to make conjectures about the sum of the three interior angles of a triangle. For example, using a paper copy of a triangle, students can tear off two angles and arrange them adjacent with each other and the third interior angle (joining the vertices) to show that they form a straight line verifying their sum is 180 degrees.

Students should have multiple opportunities to apply what they know about transformations (8.G.A.1) and angle relationships (7.G.A.2) to discover the relationship between interior and exterior angles of triangles as well as verify angle pair relationships formed when parallel lines are cut by a transversal. Students should engage in meaningful discussions to develop informal arguments and establish facts about these relationships. For example, discussing why corresponding angles are congruent using translations and why alternate interior angles are congruent using rotations.

Because students have experimented with transformations, instruction should include activities that require students to apply these understandings to informally justify the angle-angle criterion for similarity of triangles. Rather than just memorizing a rule, students should be encouraged to think about and articulate why the angle-angle postulate holds true. Using patty paper, tracing paper, manipulatives, or computer software will enhance these opportunities so that students visualize how the transformations verify the relationships. Encouraging students to think strategically and make conjectures about those relationships will help students create informal arguments to establish facts, which will be beneficial when they are expected to justify and apply these relationships in high school.

Level 4:

Students at this level of understanding should focus on effectively communicating their mathematical reasoning as facts that have been verified. Students should be able to confirm or refute conjectures about angles in triangles and angles formed when two parallel lines are cut by a transversal(s). Students should provide counterexamples with their explanation when disproving these conjectures. These critiques will help students develop a deeper understanding of angles formed with triangles and the need for clear and precise language that they will apply when justifying their reasoning.

Students should also be challenged to think about real world applications to these concepts and create and solve a real-world example of a problem that involves angles in a triangle or angles formed by parallel lines cut by a transversal.

TN Department of Education

Geometry (G)

Standard 8.G.B.3 **Cluster Heading: B. Understand and apply the Pythagorean Theorem.**

Explain a model of the Pythagorean Theorem and its converse.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize and define right triangles. Identify the hypotenuse and legs of a right triangle. Represent a number squared using a geometric representation of a square.	Determine if a triangle is a right triangle using the length of the sides. Identify which variables represent the legs and hypotenuse in the formula $a^2 + b^2 = c^2$. Construct a right triangle given the lengths of the sides.	Use a model to explain the Pythagorean Theorem. Determine if a triangle is a right triangle using the converse of the Pythagorean Theorem, given a model or side lengths.	Critique the work of others in justifying the Pythagorean Theorem. Critique the work of others in justifying the converse of the Pythagorean Theorem. Create examples of triangles and explain why they are or are not right triangles using the Pythagorean Theorem or its converse.

Instructional Focus Statements

Level 3:

In grade 7, students explored the conditions regarding side lengths and angle measures that are needed to create a triangle (7.G.A.2). In grade 8, students extend their learning to consider conditions for side lengths to create right triangles. Students should use manipulatives to explore relationships between the side lengths of right triangles and make connections to a model of the Pythagorean Theorem. Instruction should focus on having students explain the Pythagorean Theorem and then use its converse to determine if a triangle is a right triangle. It is essential that students be provided opportunities to create and explain models that illustrate the Pythagorean Theorem to discover and show that the area of the square that forms the hypotenuse is equivalent to the sum of the squares that form the legs. Students should be expected to use their understanding to give meaning to the formula $a^2 + b^2 = c^2$. For example, if the square of the longest side (c) of a triangle is equal to the sum of the squares for the other two sides (a and b), then the angle opposite of side c is a right angle and the triangle is a right triangle (the converse statement).

Students should have multiple opportunities to engage in discourse around the Pythagorean Theorem and its converse statement and explore situations that involve triangles that are and are not right triangles. Students should be encouraged to use precise mathematical language when describing right triangles. To avoid the common misconception that the Pythagorean Theorem can be used to find missing sides for non-right triangles, students need multiple experiences in testing the theorem with non-right triangles to support their understanding of how the theorem works. A thorough understanding of the Pythagorean Theorem will be important as students continue to apply this concept to solve geometric problems and justify reasoning in this and future courses.

Care must be taken when describing applications of the Pythagorean Theorem as it does have applications for non-right triangles, but students will not be expected to apply it to non-right triangles in grade 8. It is accurate to state that $a^2 + b^2$ will only be equal to c^2 in a right triangle application.

Level 4:

Students at this level should be challenged to go from informal explanations to mathematical justifications. Students should not only be challenged to make connections between representations, but also provide feedback to a peer who explains the theorem and its converse. Discussion should be facilitated so that students have opportunities to explain why a triangle is or is not a right triangle. Additionally, their peers should be encouraged to critique their explanations using precise mathematical vocabulary.

Students can also consider real-world applications for the Pythagorean Theorem and its converse and be encouraged to create their own real-world examples.

Geometry (G)

Standard 8.G.B.5 **Cluster Heading: B. Understand and apply the Pythagorean Theorem.**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the legs and the hypotenuse for any right triangle on a coordinate plane. Evaluate the square root of perfect squares.	Create a right triangle from two given points on the coordinate plane such that the two points are the endpoints of the hypotenuse, and the legs lie horizontally or vertically. Approximate the value of non-perfect square roots using a calculator or by estimating what two whole numbers it would lie between. Determine the measure of each leg for any right triangle on a coordinate plane.	Find the length of a line segment drawn on a coordinate plane using the Pythagorean Theorem. Find the distance between two points given as coordinates using the Pythagorean Theorem.	Create and solve a real-world problem that involves using the Pythagorean Theorem and the coordinate plane. Explain solution strategies using precise mathematical vocabulary. Find the distance between two coordinates using the Pythagorean Theorem without graphing.

Instructional Focus Statements

Level 3:

In grade 6, students found the distance between vertical or horizontal points on a coordinate plane (6.G.A.3). This standard extends that learning to finding the distance between two non-vertical and non-horizontal points and connects to the understanding developed in 8.G.B.4 by applying the Pythagorean

Theorem to the coordinate plane. The distance formula should not be introduced in grade 8. The focus of this standard is to build a connection between finding the length of a line segment and the Pythagorean Theorem. In high school (G.GPE.A.3/M1.GPE.A.3), students will explore connections between the Pythagorean Theorem and the distance formula.

Students should be given the opportunity to explore distance on the coordinate plane to develop the understanding that a diagonal distance on the coordinate plane is different from a horizontal or vertical distance. To avoid this common misconception, have students measure and compare a side length and diagonal of a square and connect this comparison to the square units on a coordinate plane.

Instruction should guide students to recognize that any non-vertical and non-horizontal line segment on a graph can be seen as the hypotenuse of a right triangle. Students can create a right triangle using the coordinate plane by simply drawing in the legs as horizontal and vertical line segments, whose lengths are easy to find. Students can then build on their knowledge and understanding of the Pythagorean Theorem from standard 8.G.B.4 and apply these concepts to the triangle created on the coordinate plane to find the length of the hypotenuse and thus the length of the original line segment.

When given two coordinates, instruction should lead students to plot the points on a coordinate plane and continue as above. As they work, student discourse should focus on identifying patterns that can guide them to eventually apply the Pythagorean Theorem without graphing.

It is important that students recognize that the use of the Pythagorean Theorem is a strategy for finding the length of a line segment on a coordinate plane. They should be asked to find the length of a variety of line segments including vertical, horizontal, and diagonal line segments to build an understanding that this strategy is only needed when finding the length of a diagonal line. Students should be exposed to problems where they approximate the length of an unknown side of a right triangle even when the length is not a whole number, as is often the case.

Level 4:

At this level of understanding, students should be challenged to extend their mathematical learning to real world situations where they would utilize the coordinate plane as a tool and the Pythagorean Theorem as a strategy to solve contextual problems. Students should focus on using precise mathematical language to explain their solution strategies to others.

It is important to note that the distance formula is not expected at this level. However, by identifying patterns as they work with finding the distance between two coordinates, they can develop an informal method of utilizing the Pythagorean Theorem without graphing the points. Students should be able to explain how and why their method works.

Geometry (G)

Standard 8.G.C.6 **Cluster Heading: C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

Apply the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a cone, cylinder, and sphere. Define volume as the number of unit cubes that will fill a three-dimensional object.	Find the area of a circle. Find the volume of a right prism. Identify the key dimensions (i.e., radius, height, circumference and diameter) of cones, cylinders, and spheres.	Find the volume of cones, cylinders, and spheres. Apply volume formulas to solve real-world or mathematical problems involving cones, cylinders, and spheres.	Explain how the volume formulas for a cylinder and a right prism are related. Explain how the volume formulas for a cone and a pyramid are related. Justify the relationships between the volume of a cylinder, a cone, and a sphere.

Instructional Focus Statements

Level 3:

In grade 7 (7.G.B.5), students made connections between filling a right prism and pyramid with cubic units and their volume formulas. They also explored the relationships between the area of a circle, its radius, and the number π to learn the formula for the area of a circle (7.G.B.3). In grade 8, students will combine and extend that understanding to finding the volume of cylinders, cones, and spheres. The expectation for this standard is for students to understand and apply the volume formulas for cones, cylinders, and spheres, rather than memorize them.

In grade 7, students explored filling a right prism with cubic units one layer at a time until it is full to find the volume. They connected this idea of layers to repeated addition and thus multiplication to generalize that volume of a prism can be calculated by the area of the base, regardless of the shape of the base, multiplied by its height. In grade 8, instruction should guide students to discover that this same concept also applies to finding the volume of a cylinder. Applying the volume formula for any prism, $V=Bh$, to a cylinder means that the area of the base, B, becomes πr^2. Therefore, the volume of a cylinder is $V=\pi r^2 h$. It is important to note that these formulas are equivalent and can be used interchangeably.

In grade 7, students explored the relationship between the volume of a prism with the volume of a pyramid. In grade 8, instruction should guide students to discover that the same relationship holds true between a cylinder and a cone. For example, provide a right cylinder and cone that have congruent bases and the same height and fill the cone with water or sand. Then have students predict how many times the contents of the cone can be poured into the cylinder until it's full. This will help students see that the cylinder will hold three times the volume of the cone and thus, the volume of the cone is one-third the volume of the cylinder (Cylinder: $V = Bh$ or $V = \pi r^2 h$; Cone: $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$).

As students begin to understand the volume of a sphere, they should make the connection that a sphere enclosed in a cylinder, where the diameter is equivalent to the height of the cylinder, is $\frac{2}{3}$ the volume of the cylinder. Then, by substituting the diameter of the sphere, $2r$, for the h in the formula for the volume of the cylinder, students will discover the resulting formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

It is imperative for students to discover the relationships between these solids to develop an in-depth conceptual understanding of the volume formulas. They should also apply this understanding to real-world and mathematical problems.

Level 4:

Students at this level should not only be able to apply the correct volume formulas accurately, but they should also be able to explain how they are related. They should be able to explain using precise mathematical vocabulary how the volume of a cylinder relates to the volume of a prism and how the volume of a cone relates to the volume of a pyramid. They should also be able to explain and justify the relationships between cylinders, cones, and spheres.

Additionally, students should generalize their conceptual understanding and connections of the volume formulas and use them to efficiently solve real-world and mathematical problems.

Statistics and Probability (SP)

Standard 8.SP.A.1 Cluster Heading: A. Investigate patterns of association in bivariate data.

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify any clusters and extreme values represented on the graph when given a scatterplot. Identify positive or negative association represented on the graph when given a scatterplot. Explain the meaning of the values of a given ordered pair in a context.	Determine if a scatter plot has linear, nonlinear, or no association. Identify clusters extreme values and gaps in data in a given scatter plot. Define "bivariate measurement."	Construct scatter plots using two-variable data sets. Describe patterns of association for two-variable data sets represented in scatter plots. Identify the relationship of the two quantities being represented by a scatter plot in a given context.	Create a context to describe bivariate data from a given scatter plot. Interpret a scatter plot and make predictions based on the patterns of association using precise mathematical language. Describe what clusters and outliers reveal about the data from a scatter plot.

Instructional Focus Statements

Level 3:

In grades 6 and 7, students modeled relationships between quantities the coordinate plane. Instruction should build on that understanding as students are asked to model and interpret scatterplots and describe the associations represented in their scatterplots or given scatterplots. Students should have opportunities to extend their understanding of linear relationships to investigate patterns of association between the quantities. Students should be expected to recognize and explain the meaning of positive and negative correlations, clusters, and gaps in data. Students should understand how extreme data points affect the patterns of association in a scatter plot; however, students are not expected to calculate if a value is an outlier in grade 8. An extreme value is only an "outlier" if it meets the specific definition. Students in grade 8 are not expected to calculate outliers. For this reason it is important to use precise language when dealing with extreme values, because not all extreme values are outliers. Students are only expected to recognize that a value that is “far away” from the other values in the data set may affect the center, shape and variability of the data distribution. Discourse should include opportunities to describe extreme values as deviations from associated data, as well as opportunities to describe linear, and non-linear trends. Additionally, instruction should include problems where no trends are evident. Discussions should include examples that lead students to understand that a trend in a scatter plot does not indicate cause and effect.

Through the course of learning students begin to realize that constructing a scatter plot can make it easier to identify when relationships between bivariate measurements occur. Students should also have opportunities to explore and practice constructing graphs by hand, using calculators, or through the use of computer software programs. In high school students will continue their learning of bivariate data where they will fit a function to the data and use the model to solve problems in the context of the data.

Level 4:

To deepen understanding, students at this level should be given opportunities to create contextual problems that could be used to represent a given set of data.

Opportunities for students to gather their own bivariate data for representation and analyzation should be offered for students at this level. Students could be challenged to conduct research to collect bivariate data or gather their own data and then generate a scatter plot, make predictions about the data, and summarize their findings using precise mathematical vocabulary.

Statistics and Probability (SP)

Standard 8.SP.A.2 **Cluster Heading: A. Investigate patterns of association in bivariate data.**

Know that straight lines are widely used to model linear relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify features of a linear relationship. Determine if a scatter plot has linear association.	Determine if a scatter plot has positive, negative, or no relationship between two quantities. Draw a straight line that closely fits the data points on a scatter plot.	Construct a table of values, plot points, and draw a straight line that closely fits the data points to model linear relationships in context. Determine which line most closely models the association of the data when given a scatter plot with various possible lines of fit. Determine the accuracy of a line of fit based on the closeness of the data points to the line.	Determine the mean of the x-values and y-values on a scatter plot to identify the centroid point on the estimated model line of fit. Explain the meaning of the line of fit and its properties using precise mathematical language in terms of the context. Critique the accuracy of a line of fit to a data set represented in a scatterplot.

Instructional Focus Statements

Level 3:

In standard 8.SP.A.1, students are expected to describe qualitative bivariate data that is represented on a scatter plot. From that standard, essential vocabulary terms such as outliers, clusters, positive, negative, strong, weak, linear correlation, and non-linear correlation are used to describe key features of scatter plots. As a progression, students now determine when a linear relationship is present in a scatter plot and explain the relationship if one exists. Students should be exposed to strong and weak examples of linear and non-linear relationships and engage in discourse around the fit of the line.

Discussion of a variety of graphs should lead students to the understanding that a straight line can model a relationship of the data points when there appears to be a linear association. When a linear relationship exists, students should realize that the closer the points are to a line, the stronger the linear relationship. Students might struggle to draw a straight line, so a ruler or other straight edge could be used to help students decide where to place their lines. Transparent rulers could be beneficial for students because they can see the points while finding the best place for their line. When students initially begin fitting lines to data, instruction could begin by providing them with pre-plotted scatter plots so that the focus is on drawing the line based on the trend represented by the data points.

Students should have opportunities to compare lines of fit when given the same set of data. A common misconception for students is thinking that their lines of fit for the same set of data will be or must be exactly the same, not understanding that the lines of fit are informally drawn to approximate an equation and represent an association between two data sets. Instruction should facilitate open discussion with students and give opportunity for students to present multiple lines of fit and compare all options. This gives the opportunity to clarify any informal understandings that students may apply in the construction of the line and solidify the understanding that the most appropriate line is the one that comes closest to most data points, and therefore the line may or may not go through all or any of the data points.

Level 4:

To deepen understanding, students at this level should be given opportunities to create contextual problems that could be used to represent a given set of data.

Opportunities for students to gather their own bivariate data for representation and analyzation should be offered for students at this level. Students could be challenged to conduct research to collect bivariate data or gather their own data and then generate a scatter plot, make predictions about the data, and summarize their findings using precise mathematical vocabulary.

Statistics and Probability (SP)

Standard 8.SP.A.3 **Cluster Heading: A. Investigate patterns of association in bivariate data.**

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercepts. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the slope and the *y*-intercept for a line that is graphed on a coordinate plane. Describe the relationship between variables on a graph in context. Write the equation of a line given the slope and y-intercept.	Draw a line of fit for bivariate measurement data. Write an equation for the line of fit on a scatter plot.	Use a linear model to solve contextual problems. Interpret the slope of a linear model in context of bivariate measurement data. Interpret the *y*-intercept of a linear model in context of bivariate measurement data.	Make predictions using a linear model and describe the accuracy of the predictions in the given context. Conduct an experiment to gather, graph, and write a linear equation for bivariate data. Assess the reasonableness of using the point on a linear graph in the context of the situation.

Instructional Focus Statements

Level 3:

Students extend their understanding from grades 6 and 7 where they graphed and created equations of quantitative relationships. Additionally, students interpreted the meaning of points (x, y) and rates within proportional relationships and eventually applied this same understanding to non-proportional relationships.

Instruction should build on students' experience of modeling linear relationships by constructing scatter plots and lead them to solve authentic problems involving bivariate data using linear equations. In addition to writing equations that represent the line fit to the data, discussion should go beyond identifying and should focus on the meaning of the slope and y-intercept of the line in terms of the context. Students should be exposed to a variety of scatter plots, where the points are coplanar, but generally non-collinear, and be able to select appropriate points in order to write the equation of the line. A variety contextual questions should be presented to allow students to realize that the equation of the line of fit can be used to solve these problems. Discourse should include opportunities for students to explain how they determined the equation of the line and how closely values found using the equation are to the actual values in the given data set. A solid understanding of linear models will be essential as students move beyond linear models and explore quadratic and exponential models in high school.

Level 4:

At this level of understanding, students can be challenged to take on more ownership of the entire process of analyzing bivariate data. Opportunities should be given for students to hypothesize a linear relationship, gathering their own data, creating their own graph and line of fit, and writing an equation for the linear association they have found in their experiment. Additional elements of these activities could include giving a presentation that describes and interprets the slope and intercept for others as they present their findings, or giving students the opportunity to interpret the slope and y-intercept represented in the linear models of others. Students should be able to debate the reasonableness of some points that may fit the line and justify why these points would not make sense due to the context of the situation. For example, although a point on a line may represent 3.6 this would not make sense if our data were about people, whereas it would make sense if we were talking about a measurement.

To further prepare students for models of other functions, instruction can also lead students to use the line of fit to make predictions for what is likely to happen in a given situation. Students should be encouraged to analyze their own predications and critique the predications of others for reasonableness in terms of the contexts given.

TN Department of Education

Statistics and Probability (SP)

Standard 8.SP.B.4 Cluster Heading: B. Investigate chance processes and develop, use, and evaluate probability models.
Find probabilities of and represent sample spaces for compound events using organized lists, tables, tree diagrams, and simulation.
8.SP.B.4.a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
8.SP.B.4.b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the probability of a simple event. Understand that a probability is between 0 and 1, and that something unlikely to happen has a probability closer to 0, something likely to happen has a probability closer to 1, and something equally likely to happen is halfway between 0 and 1. Express the probability of a single event using the appropriate terms impossible, unlikely, equally likely, likely, or certain.	Express probability of a simple event as a fraction, decimal and/or percent. Differentiate between simple events and compound events. Differentiate between theoretical and experimental probability. Estimate the probability of an event by collecting data and observing its long-run frequency.	Determine the sample space of a compound event. Use probabilities to make decisions in real-world situations. Determine the probability of a compound event from a sample space. Recognize that the number of possible outcomes for a compound event is determined by multiplying the number of outcomes for each individual event. Determine the probability of compound events using lists, tables,	Determine whether or not a given probability model is plausible and justify your explanation. Explain how a given contextual situation models a compound event and how the probability can be approximated. Design a simulation and use the results to estimate a probability.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		tree diagrams, and simulations. Compare compound probabilities that are based on theoretical models with experimental probability simulations. Express the probability of a compound event as a fraction, decimal, and/or percent.	

Instructional Focus Statements

Level 3:

In grade 7, students explored the probability of simple events by developing models and comparing them with experimental probability (standards 7.SP.C.6 and 7.SP.C.7). Students in grade 8, will build on their knowledge of single events to determine the probability of compound events. As they did with simple events, students should have opportunities to conduct experiments using a variety of random generation devices (e.g., spinners, number cubes, coin toss, etc.).

Instruction should include opportunities to interpret and create probability models that illustrate possible outcomes and sample space for compound events. Visual models will be helpful as some students might be tempted to add, rather than multiply, when finding the probability of these events. Opportunities for discourse will allow students to compare simple events to compound events and explain, both orally and in writing, similarities and differences between the two using appropriate probability terms. Students should be given opportunities to discover and understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Instruction should lead students to recognize the benefits of using organized lists, tables, and tree diagrams when representing sample spaces for compound events.

Level 4:

Students at this level should go beyond calculating probabilities, to using probabilities to make informed decisions about real-world situations. Students should design their own simulations and use the results to estimate and make theoretical predications based on their experiments. Students should also be able to explain using precise mathematical vocabulary why or why not a given probability model is valid based on the context.