Seventh Grade Mathematics
Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the "how" and the "why" of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase "procedural skill and fluency" is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the "why" behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

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Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:
The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:
Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

Ratios and Proportional Relationships (RP)

Standard 7.RP.A.1 Cluster Heading: A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
For example, if a person walks 1/2 mile in each15 minutes, compute the unit rate as the complex fraction (1/2) / (1/4) miles per hour, equivalently 2 miles per hour.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a ratio as a unit rate when it is expressed in the form $a : 1$ or $1 : a$. Choose a unit rate to represent a ratio presented as a complex fraction given a visual model. Choose a unit rate to model situations involving whole number quantities.	Choose a unit rate to model situations involving fractional quantities presented as a non-simplified complex fraction. Write ratios to model situations involving whole number quantities measured in like or different units and use the ratio to determine a unit rate.	Interpret a complex fraction as the division of two fractions. Calculate unit rates from a given complex fraction or ratio with like or unlike units. Write ratios to model real-world situations involving fractional quantities measured in like or different units and use the ratio to determine a unit rate.	Write ratios that model situations involving fractional quantities measured in like or different units, use the ratio to determine a unit rate, and explain the unit rate in terms of the context of the situation. Create contextual problems that model situations involving fractional quantities measured in like or different units and provide the unit rate solution.

Instructional Focus Statements

Level 3:

The focus of this standard is extending a students' understanding of unit rates developed in grade 6 (6.RP.A.2) to computing unit rates using ratios from fractional quantities known as complex fractions. Students developed an understanding of quotients of fractions in grade 6 (6.NS.A.1). In grade 7, students will utilize this understanding to compute unit rates from fractional quantities. Students should discover that computing unit rates with fractions is the same concept as unit rates with whole numbers. To enhance the foundational understanding of ratios as division of fractions, students should be

encouraged to use visual representations (bar models, double number lines and ratio tables, etc.) or manipulatives (fraction tiles, pattern blocks, etc.) to model the fractional quantities in order to compute a unit rate. Students may have misconceptions when writing a ratio in the form of a complex fraction. It is important for students to understand that the fraction bar is the same as the common division symbol. Also, students should make connections to visual models to grasp the fractional unit rate. Using the example from the standard, students should utilize a visual representation (double number line, bar model, etc.) to see that if a person walks 1/2 mile every 1/4 hour, results in 4 of the 1/2 miles in a total hour, which is equivalently a unit rate of 2 miles per hour. Students should explain their reasoning about computing unit rates with complex fractions with like and different units, including identifying when errors can be presented.

Level 4:

Students should extend their understanding of computing unit rates associated with ratios of fractions to make connections between visual representations and verbal descriptions. This should be explained by using precise mathematical language. Students should also be well versed in working with like and different units and make sense of contextual problems (MP1) to determine when errors or misconceptions could be made. Additionally, students be able to translate their understanding to create contextual problems that model situations involving fractional quantities measured in like or different units and use the ratio to determine a unit rate.

Ratios and Proportional Relationships (RP)

Standard 7.RP.A.2 Cluster Heading: A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
Recognize and represent proportional relationships between quantities.
7.RP.A.2a Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).
7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2c Use the concept of equality to represent proportional relationships with equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as: $t = pn$.
7.RP.A.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Graph points on the coordinate plane when given a table of values. From context generate tables of equivalent ratios, find a missing number in the table, and use the table to plot the ratios on a coordinate graph. Choose the constant of proportionality (unit rate) when given a graph. Differentiate between independent and dependent variables.	Know that the graph of a proportional relationship must be linear and pass through the origin. Choose the constant of proportionality (unit rate) when given a table. Identify the constant of proportionality (unit rate) as the constant factor that is multiplied by one quantity to get the other when given a table of equivalent ratios.	Generate a graph and use it to determine if quantities are proportionally related. Generate a table of values and use it to determine if quantities are proportionally related. Determine if the quantities are proportionally related when given a verbal description or diagram. Identify the constant of proportionality (unit rate) from a wide variety of representations of a	Represent a non-proportional relationship and explain why the quantities are not proportionally related. Provide a contextual situation to represent a given a table, graph, or equation. Explain the constant of proportionality (unit rate) in terms of the context. Use the constant of proportionality

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the constant of proportionality (unit rate) when given an equation in the form $y = kx$.	Recognize that x is the input and y is the output in the equation $y = kx$. Use the constant of proportionality (unit rate) to write an equation in the form of $y = kx$, where k is the constant of proportionality when given a table of values.	proportional relationship. Use the concept of equality to write a two-variable equation (in the form of $y = kx$) to represent a proportional relationship when given a context. Explain what the point (1, r) means in terms of the situation represented when given a graph. Interpret the meaning of any ordered pair on a graph in terms of the context.	to make connections between multiple representations in terms of the context. Draw conclusions that all points on a line are proportional to all other points on the line by relying on tables, verbal statements, or logical arguments to draw the conclusion. Explain why graphs of proportional relationships go through the origin.

Instructional Focus Statements

Level 3:

In grade 6, students understand the concept of ratios (6.RP.A.1) and unit rates (6.RP.A.2) and use them to solve problems (6.RP.A.3). Instruction for 7.RP.A.2 should build on students' understanding of equivalent ratios and unit rates to recognize and represent proportional relationships. A proportion is a relationship between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change. Students should decide if quantities are in a proportional relationship by determining if ratios in a table are equivalent. Another method is to graph the relationship on a coordinate plane and observe whether the graph is a straight line that goes through the point of origin (0,0). To develop a solid understanding of proportional relationships, students should be exposed to both proportional and non-proportional situations and engage in discussions where they make generalizations about proportional representations.

Students should be able to identify the constant of proportionality from multiple representations, including tables, graphs, equations, diagrams, and verbal descriptions. Students should understand that the constant of proportionality is one unit to a specified quantity or vice versa. When analyzing a table of values, students should engage in discourse where they determine whether a constant of proportionality (unit rate) exists. To encourage students

to consider the constant factor rather than an additive pattern, teachers should minimize the use of consecutive values in tables. Students should discover that there is a multiplicative relationship between the quantities and that the constant of proportionality can also be determined through division. Examples should be given in multiple representations and discussion should lead students to various methods of reasoning to make connections between the various representations.

From a proportional relationship represented as a table, graph or verbal description, students should notice that the *y*-coordinate is always a multiple of the *x*-coordinate. Using the concept of equality, students should represent the proportional relationship with an equation in the form $y = kx$, Students should notice that *x* is the value of the independent variable, k is the constant of proportionality (unit rate) and *y* is the value of the dependent variable. Although *x* and *y* are commonly used variables, using variables that relate to the context could prevent students from reversing the variables. Students could also benefit from hearing the relationship verbally stated. For example, rather than saying *p = $4t*, a student would say "The profit earned is $4 times the price of each ticket". Students would be less likely to express the relationship as *t = $4p*. When providing context, students should practice with situations where "for every" and "for each" are not explicitly stated. Doing so will promote rich conversations about interpreting the context to determine the relationship between two quantities. Students should also be able to recognize that equations involving addition or subtracting, such as $y = ax + 5$, are not proportional because there is not a constant a that can be multiplied by x to get y.

Because a constant of proportionality (unit rate) is always involved in proportional situations, students should discover that a graph of a proportional relationship always goes through (1.*r*), where r is the unit rate. When given problems with a real-world context, students should be able to explain the meaning of any point on the graph in the content of the situation. For example, point (1, 20) represents the constant of proportional (unit rate) and means there is 1 teacher for every 20 students at the school. Students should also explain the meaning of the point (0,0) in context and understand why points in Quadrant III might not be appropriate for the given real-world context.

Note that computation using cross-multiplication is not a part of this standard. Also note that in grade 7, students should not focus on the slope formula, but rather the relationship between the quantities when graphing quantities that are proportionally related. This understanding will be beneficial when they encounter problems involving scale drawings (7.G.A.1) of geometric figures as they will then associate the constant of proportionality with scale factor.

Level 4:

Students at this level should have opportunities to create situations and representations of both proportional and non-proportional relationships and explain whether the quantities are proportionally related. As students solidify their understanding, they should be able to identify and explain unit rates in multiple representations when the unit rate is represented in a form other than one to a quantity by employing their knowledge of proportional relationships and prior knowledge of equivalent fractions and explain their reasoning. Although students can choose how they will represent these relationships, discussion should be facilitated to encourage the connection between the representations. Discussions should focus on the constant of

proportionality in terms of the context and how it is the same for all representations. Students should extend their understanding of identifying unit rates in multiple representations to explain the connections between the unit rate and the verbal context interchangeably. Additionally, students should make connections to both unit rates of (1, y) and (x, 1) to the context as well as the connection to one another using precise mathematical vocabulary.

At this level, teachers should encourage students to go beyond determining if a graph represents a proportional relationship by asking them to also be able to explain why the graphs of proportional relationships must be a straight line through the origin. This can be discovered by discussing graphs of proportional and non-proportional relationships and pressing students to articulate how the relationships differ. Students should be challenged to represent non-proportional relationships in other ways, including context.

Ratios and Proportional Relationships (RP)

Standard 7.RP.A.3 Cluster Heading: A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
Use proportional relationships to solve multi-step ratio and percent problems. *Examples: batting averages, recipes, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error, etc.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand that a percent is a ratio comparing a number to 100. Choose a proportion that models a simple mathematical or real-world situation involving a ratio or percent.	Solve simple ratio or percent problems without a context. Solve percent problems involving both finding a percent of a quantity and finding the whole.	Solve multi-step ratio and percent problems in a variety of real-world contexts. Determine the percent change from one quantity to another and identify the change as an increase or a decrease.	Solve multi-step ratio and percent problems in complex situations and explain the solution in terms of the context.

Instructional Focus Statements

Level 3:

In grade 6, students learn that a percent is a specific type of ratio where the second quantity is 100 and solve percent problems involving both finding a precent of a quantity and finding a whole (6.RP.A.3). In grade 7, students should extend understanding of proportional reasoning to solve multi-step ratio and percent problems using visual and algebraic models. For example, students should use bar models, double number lines and ratio tables to help identify how the values of the problem are related and use equations to solve problems involving simple interest, sale prices, tax, etc. Encourage students to explain their work using a variety of representations (numbers, words, pictures, physical objects, or equations). Students should draw on their experience solving contextual problems to make sense of multi-step ratio and precent problems and persevere in solving them (MP1).

Allow students to think about everyday situations involving percent and discuss if their ideas show a markup, markdown, sale price, simple interest, tax, gratuity, commission, etc. Students should understand that different situations such as tax, tip, interest and commission are similar because the total is

the original amount with an additional amount added on. Students should understand the precise mathematical vocabulary within the examples of the standard in contextual situations and be able to explain how they are used in each situation. As students work with percentages, they should understand that problems involving percent increase or percent decrease require careful attention to the referent whole. Students should start with identifying the whole/original amount of the percent being defined to understand the meaning of the percent of increase/decrease.

Students should develop an in-depth, conceptual understanding of generating equivalent ratios and proportional reasoning to apply that to the contextual problem. Using cross multiplication to solve problems involving proportional relationships does not lend itself to building a strong conceptual understanding.

Level 4:

As students extend their understanding of multi-step ratio and percent problems, they should be able to extract essential information from more complex contextual problems. Students should also be able to determine efficient solutions paths to solve ratio and percent problems and explain why they have chosen the selected solution path. Additionally, students should be able to use precise mathematical vocabulary in their explanations and its connection to the contextual situation.

The Number System (NS)

Standard 7.NS.A.1 Cluster Heading: A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.1a Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.A.1b Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.

7.NS.A.1c Apply properties of operations as strategies to add and subtract rational numbers.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify that the sum of a number and its opposite is 0, and that these numbers are called “additive inverses”.	Identify situations in which opposite quantities combine to make 0. Choose a number line diagram that represents a given addition or subtraction problem of rational numbers. Choose a real-world context to represent a given sum or difference of rational numbers.	Generate a number line diagram that represents a given addition or subtraction problem of rational numbers. Add and subtract rational numbers. Explain that the sum $p + q$ is located a distance $\|q\|$ from p and when it goes in the negative direction and when it goes in the positive direction.	Represent addition and subtraction of rational numbers on a horizontal and a vertical number line. Provide a model and an explanation to show why $p - q$ and $p + (-q)$ are equal. Create contextual problems that involve applying properties of operations to add and subtract rational numbers and interpret the respective sum or difference.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Create a real-world context to represent a given sum or difference of rational numbers. Identify that the distance between two rational numbers on a number line is the absolute value of their difference.	

Instructional Focus Statements

Level 3:

Instruction should focus on students developing an understanding of and computing with rational numbers, including integers, positive and negative fractions, and positive and negative decimals. Students should encounter situations promoting positive and negative numbers combining to make zero in real-world situations and model this situation on both vertical and horizontal number lines. Foundational to this is a student's ability to see real-world examples in terms of positive and negative numbers such as 30 degrees below zero is represented as -30°. Modeling signed numbers on a number line enhances the idea of how a certain number of moves in a positive direction from zero combined with the same number of moves in the opposite direction ends at zero. Students should also understand that these values are additive inverses because their sum is zero.

As students become more confident in working with additive inverses to make zero, they should use this technique to develop an understanding of how to add and subtract rational numbers using number lines, counters, and other visual representations. It is important that students are afforded the opportunity to develop this conceptual understanding as opposed to being presented with a set of "rules" for adding and subtracting signed numbers. Specifically, with subtraction of rational numbers, one approach should be thinking of subtraction in terms of addition using the additive inverse. This results in defining the distance between two rational numbers on a number line as the absolute value of the difference between the two numbers. Students should then progress to discover and apply formal rules for adding and subtracting rational numbers with equations and support their reasoning with verbal and written explanations accompanied by visual models. In previous grade-levels students used the commutative, associate, and additive identity properties with whole numbers. Students should also be exposed to applying these properties when working with rational numbers. As students solidify their understanding of applying operations as strategies to add and subtract rational numbers, they should be able to solve real-world problems providing verbal and written justifications using mathematical language.

Level 4:

Students should be able to find the sum or difference of rational numbers in more complex equations. Additionally, when working in real-world situations, problems should be posed eliciting the unknown in any position and where multiple properties are embedded. To further extend understanding, students should reason about contextual problems that involve applying properties of operations to add and subtract rational numbers and interpret the respective sum or difference. This interpretation should be explained using precise mathematical language and supported by visual representations.

The Number System (NS)

Standard 7.NS.A.2 Cluster Heading: A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.A.2a Understand that multiplication is extended from fractions to all rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates or eventually repeats.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Convert a fraction with a denominator that is a multiple of 10 to a decimal. Divide multi-digit numbers by applying properties of operations as strategies.	Convert fractions with a denominator of 2, 3, 4, 5, 6, 8, or 10 to a decimal. Divide multi-digit numbers using long division. Choose a real-world context to represent a given product or quotient of rational numbers.	Multiply and divide a wide variety of rational numbers. Interpret products and quotients of rational numbers by describing real-world contexts. Apply properties of operations as strategies to multiply and divide rational numbers. Create a real-world context to represent a given product or	Create problems that involve multiplying and dividing rational numbers in real-world and mathematical problems and interpret the respective product and quotient. Make generalizations about long division quotients to determine if the decimal form of the rational number will repeat or terminate. Explain why multiplying or dividing

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		quotient of rational numbers. Convert a rational number to a decimal using long division; know that the decimal form of rational numbers terminates or eventually repeats.	a negative number by a negative number results in a positive answer using appropriate mathematical vocabulary. Explain why multiplying or dividing a negative number by a positive number or vice versa results in a negative answer using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

Previously students have used visual models to represent addition and subtraction of rational numbers. When students begin to develop an understanding of multiplication and division of rational numbers, students must rely increasingly on their understanding of both the properties of operations and their knowledge of addition and subtraction with rational numbers. In order to build the necessary bridges from their previous understandings to situations where one or more of the numbers might be negative, they may rely on repeated addition or repeated subtraction in order to discover the relationships that exist when multiplying and dividing with signed numbers. Additionally, using real-world examples strengthens the conceptual understanding of multiplication and division of rational numbers. For example, students can understand that if 3 people owe a debt of $5, then the total debt owed is $15 resulting in the equation 3 • -5 = -15. Connecting multiplication to repeated addition can also support students in understanding the concept of multiplication with signed rational numbers. In the same example, 3 • -5 = 15, can be represented as repeated addition as (-5) + (-5) + (-5) = -15. Students should employ their understanding that a negative number in terms of the opposite of a number, as learned in grade 6 (6.NS.C.5), and the use of the distributive property to deduct that (-1)(-1) can be represented as – ((1)(-1)) =1.

When students are presented with division of signed rational numbers, they should make the connection that multiplication and division are inverse operations. In the example, 3 x -5 = -15, students should also know that the same is true for the division equation of -15 ÷ 3 = -5 or $\frac{-15}{3} = -5$. Students should interchangeably use notation for division as p ÷ q and $\frac{p}{q}$ and understand that division by zero is not defined. Students should use these connections to discover the rules for multiplying and dividing rational numbers. To extend students understanding of rational number division, students

should use long division to convert rational numbers in fraction from to decimal form. Additionally, students should be able to sort the decimal form of rational numbers into two types: terminating or repeating.

Level 4:

As students solidify their conceptual understanding of multiplication and division of rational numbers, they should be able to explain the relationship between the inverse operations. Students should also be able to explain in verbal and written form the connections between multiplication and division in real-world contexts explain how they discover a set of rules for signed numbers. Additionally, students should extend their understanding of converting a fraction to decimal form by making generalizations about long division quotients to determine if the decimal form of the rational number will repeat or terminate. This should be supported with verbal and written justification using precise mathematical language.

The Number System (NS)

Standard 7.NS.A.3 Cluster Heading: A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve one-step mathematical problems involving addition and subtraction of positive rational numbers. Solve one-step real-world problems involving addition and subtraction of positive rational numbers. Apply the order of operations to evaluate numerical expressions with positive rational numbers. Choose a division problem that represents a complex fraction.	Solve two-step mathematical problems involving multiplication and division of positive rational numbers. Solve two-step real-world problems involving multiplication and division of positive rational numbers. Rewrite a complex fraction as a division problem. Apply the order of operations to evaluate numerical expressions with rational numbers and exponents.	Solve multi-step real-world problems involving the four operations with rational numbers. Solve mathematical problems involving operations with complex fractions. Solve real-world problems involving operations with complex fractions. Apply the order of operations (including grouping symbols and exponents) to solve mathematical and real-world situations involving rational numbers.	Create multi-step real-world problems that involve a wide variety of operations with rational numbers. Justify the solution path to solve real-world problems involving multiple operations with rational numbers. Explain the solution to a real-world problem in terms of its context.

TN Department of Education

Instructional Focus Statements

Level 3:

In grade 6 (6.EE.A.2c), students used the order of operations to evaluate numerical expressions involving positive rational numbers. In standards 7.NS.A.1 and 7.NS.A.2, students are developing a conceptual understanding of operations with rational numbers, including integers, positive and negative fractions, and positive and negative decimals. In standard 7.NS.A.3 students should extend this understanding to apply the four operations with rational numbers to solve real-world and mathematical problems.

Students should have multiple opportunities to interpret and write expressions and equations to represent the context of a wide variety of real-world problems. Problems should require multiple operations as well as positive and negative rational numbers, including fractions, decimals and integers. A variety of problems that require grouping symbols and continue to use the use of the distributive, commutative, associative, and additive identity properties should be included throughout instruction to help students learn to justify their manipulation of numbers. Students should be asked to explain why they chose the operations used in expressions and continue to use the order of operations to accurately evaluate those expressions. Students should be given opportunities to discuss and critique the reasoning of others. Classroom discourse should focus on what is happening in the context, rather than associating key words with certain operations which can sometimes be misleading. Students should be able to interpret the real-world problem in order to determine which operation(s) are necessary to solve the problem, including using models and other representations.

Problems that require division of rational numbers should include complex fractions, $\frac{a}{b}$, where a and b can be fractions. Students should interpret a complex faction as the division of two fractions. Previously, students made the connection that multiplication and division are inverse operations when manipulating integers. As students extend this understanding, they should have opportunities to discover that the relationship remains true with complex fractions. For example, $\frac{1/2}{1/4} = 2$ and $2 \times 1/4 = 1/2$. This learning will lead to an understanding of ratios made up of fractions in grade 7 (7.RP.A.1). Solidifying the understanding of operations with a wide variety of rational numbers will be essential as students explore algebraic concepts in grade 8.

Level 4:

At this level, students should demonstrate a strong conceptual understanding of performing operations with rational numbers. Students should be given opportunities to create and solve multi-step, real-world problems that involve positive and negative rational numbers. Classroom discourse should focus on the meaning of the solution in terms of the context and the reasonableness of their solutions. Students should support their solution paths in multiple ways and fully explain all steps in the problem-solving process. A student who chooses to use a tape diagram to illustrate the problem-solving process might employ both a part-whole model and a comparative model to justify multiple operations required to arrive at an answer. Additionally, students should be challenged to consider alternative methods that are mathematically sound, which yield the same solution. In addition to finding solutions to these problems, students should be posed with questions that solicit an explanation of their solutions in terms of the context. These explanations should be presented using precise mathematical language.

Expressions and Equations (EE)

Standard 7.EE.A.1 Cluster Heading: A. Use properties of operations to generate equivalent expressions.

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract linear expressions with positive rational coefficients. Choose equivalent expressions resulting from factoring or expanding linear expressions with positive whole number coefficients.	Choose equivalent expressions resulting from adding, subtracting, factoring, or expanding linear expressions with rational coefficients.	Add, subtract, factor, or expand linear expressions with rational coefficients to generate equivalent expressions.	Generate equivalent expressions resulting from adding, subtracting, factoring, or expanding linear expressions with rational coefficients and explain their reasoning using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

This standard is a continuation of coursework previously completed in 6.EE.A.3 and 6.EE.A.4. In grade 6, students apply the properties of operations to generate equivalent expressions. For example, students apply the distributive property to the expression 3(2 + x) resulting in 6 + 3x and y + y + y in resulting in 3y to produce equivalent expressions. Students also identified when expressions are equivalent using the substitution property. In grade 7, students are introduced to operations with positive and negative integers (7.NS.A.1 and 7.NS.A.2) and should employ this understanding and their prior knowledge of properties of operations (distributive, commutative, associative, identity, and inverse properties) of addition and multiplication as strategies for adding, subtracting, factoring, and expanding linear expressions. When applying properties, it is imperative that students have a strong conceptual understanding of using the order of operations when producing equivalent linear expressions.

As students are solidifying their understanding of the properties of operations, it is important that they can recognize and apply the properties to expressions. For example, students should recognize that (3 + 2) + 4 = 4 + (3 + 2) is an example of the commutative property. Likewise, students should

recognize that both 2 x (4 + 1) = 2 x 4 + 2 x 1 and 3 x 7 + 3 x 5 = 3 x (7 + 5) are examples of the distributive property. A solid understanding of these properties is foundational for concepts such as factoring polynomials in high school courses.

Allow students to reason mathematically to identify equivalent expressions. For example, *"Lisa thinks the two expressions 4(2a – 3) + 3a and 11a – 3 are equivalent? Is she correct? Explain your reasoning?"* Students might reason using properties or reason using substitution to determine that Lisa is not correct. Facilitate discourse that asks students to defend why expressions are or are not equivalent. Provide opportunities for students to discover that there can be more than one expression equivalent to a given expression. For example, what expression can be added to 5x+7 to make it equivalent to 9(x+1)?

In this standard, coefficients can include rational numbers that encompass integers, positive/negative fractions, and decimals. Instruction for this standard should emphasize the use of properties of operations and substitution to identity equivalent expressions, as opposed to the process of simplifying expressions. Concrete manipulatives such as colored squares or algebra tiles can help students visualize various forms of equivalent expressions. This standard reinforces foundational skills for grade-level coursework in which students rewrite expressions to reveal specific quantities (7.EE.A.2) and will lead to future coursework in which students will work with expression types other than linear expressions.

Level 4:

Students should develop an in-depth understanding of applying properties of operations and be flexible in selecting the most efficient operations to use and explain their reasoning using precise mathematical vocabulary. Students should also be able to justify their solutions by substituting values into equivalent expressions to verify that both expressions result in the same value and are therefore equivalent expressions, and their solution path is correct. Additionally, students should extend their knowledge of applying properties for operations as strategies to add, subtract, factor, and expand liner expressions as a part of solving contextual problems. This will be a foundational skill for future coursework in which students will work with other expression types.

Equations and Expressions (EE)

Standard 7.EE.A.2 Cluster Heading: A. Use properties of operations to generate equivalent expressions.

Rewrite and connect equivalent expressions in different forms in a contextual problem to provide multiple ways of interpreting the problem and investigating how the quantities in it are related. *For example, shoes are on sale at a 25% discount. How is the discounted price P related to the original cost C of the shoes? C - .25C = P. In other words, P is 75% of the original cost for C - .25C can be written as .75C.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an expression that represents a given real-world situation. Identify independent and dependent variables in a contextual problem.	Choose a form of an expression that accurately highlights a particular relationship between quantities in a problem.	Generate an equivalent expression to highlight a given relationship within the problem, given a contextual problem and an expression that represents it. Choose multiple forms of an expression and explain how each accurately highlights a particular relationship between quantities in a problem.	Generate an expression that accurately highlights a particular relationship between quantities in a contextual problem. Explain the relationship that exist between different expressions representing the same contextual problem and write a justification as to when each might be more beneficial.

Instructional Focus Statements

Level 3:

Students are developing a foundational understanding that rewriting an equivalent expression can show the relationship among the terms in an expression. In grade 6 (6.EE.A.2), students used mathematical vocabulary to identify and describe parts of an expression. In grade 7, students interpret parts of an expression, such as the coefficient, constant, term, and variable, based on the context of the problem. By initially using simple contexts that have accessible entry points, students will conceptually develop an understanding of the connections that exist between the context and various forms of

equivalent expressions. Encourage students to draw and label visual representations of contextual problems. Facilitate discussion that allows students to investigate the connection between parts of an expression and their drawing and the connections between the structure of each expression (MP7). For example, students might explore that there are various equivalent expressions that could be written to represent the perimeter of a rectangle.

Students should be presented with real-world problems that can be modeled with more than one expression. While equivalent expressions describe the same situation, engage students in discussion about how different expressions can give different perspectives and might be more useful for certain purposes. For example, an increase of 5% might be written as the expression $a + 0.5a$ or the equivalent expression $1.05a$. It is important for students to see that rewriting an expression into the smallest length or "simplest" form is not always advantageous.

Level 4:

Students should be able to generate an expression resulting from a real-world problem and explain the parts of the expressions in terms of the context. Students should also be able to generate multiple equivalent expressions and justify the use of one over the other. Additionally, they should be able to provide an explanation of the connection that exists between the resultant expressions. Justifications and explanations should be in both verbal and written form using precise mathematical vocabulary.

Equations and Expressions (EE)

Standard 7.EE.B.3 Cluster Heading: B. Solve real-world and mathematical problems using numerical and algebraic expressions and equations and inequalities.

Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers presented in any form (whole numbers, fractions, and decimals).

7.EE.B.3a Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate.

7.EE.B.3b Assess the reasonableness of answers using mental computation and estimation strategies.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve single-step mathematical problems with positive rational numbers presented in a single form. Conversion between forms of rational numbers is not required to solve the problem.	Solve single-step mathematical problems involving positive and negative rational numbers written in a single form. Conversion between forms of rational numbers is not required to solve the problem.	Solve multi-step, real-world and mathematical problems with rational numbers where conversion between forms of rational numbers may or may not be required. Apply appropriate estimation strategies to solve multi-step real-word and mathematical problems.	Solve multi-step, real-world and mathematical problems with rational numbers where conversion between forms of rational numbers is required. Assess the reasonableness of answers resulting from multi-step, real-world and mathematical problems with rational numbers presented in any multiple forms.

Instructional Focus Statements

Level 3:

Students should solve multi-step real-world and mathematical problems that contain integers, fractions, and decimals. Students should also use previously acquired skills around converting fractions, decimals, and percentages and use properties of operations to find equivalent forms of expressions when needed. Students should solidify their understanding by checking their solutions for reasonableness using estimation strategies, for example:

- rounding values, then adjusting the estimate to take into account remaining amounts,
- using compatible numbers to allow for common factors for multiplication and division, or addition and subtraction with larger place values,
- using benchmark numbers such as using 3 for $2\frac{7}{8}$ to make an estimate.

When solving multi-step real-world and mathematical problems, allow students the opportunity to make sense of problems and persevere in solving them (MP1). Mathematically proficient students start solving a problem by explaining to themselves the meaning of a situation and looking for entry points to its solution rather than relying on key words. Depending on the context of the problem, students might use concrete manipulatives, draw a picture, use number lines, or create a bar model to represent and make sense of the situation. Encourage students to check their answers using a different method and continually ask themselves, *"Does this make sense? Is my answer reasonable?"*

Level 4:

Students should extend their understanding of solving multi-step real-world and mathematical problems by assessing solutions for reasonableness when working with rational numbers presented in any form. Additionally, they should be able to provide an explanation of their solution path, reasoning for their estimation strategy, and how their solution connects to the contextual problem. Justifications and explanations should be in both verbal and written form using precise mathematical vocabulary.

Equations and Expressions (EE)

Standard 7.EE.B.4 Cluster Heading: B. Solve real-world and mathematical problems using numerical and algebraic expressions and equations and inequalities.

Use variables to represent quantities in a real-world and mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.B.4a Solve real-world and mathematical problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

7.EE.B.4b Solve real-world and mathematical problems leading to inequalities of the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ where p, q, and r are specific rational numbers. Graph the solution set of the inequality on a number line and interpret it in the context of the problem. *(Note that inequalities using >, <, ≥, and ≤ are included in this standard.) For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the variable quantity in a real-world or mathematical situation. Choose an equation in the form $p + q = r$ or $px = r$ to represent a given contextual problem. Choose the solution for equations of the form $px + q = r$ and $p(x + q) = r$ when p, q, and x are all whole numbers.	Choose an equation in the form $px + q = r$ or $p(x + q) = r$ to represent a given contextual problem. Choose a number line graph that represents the solution set for an inequality in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$. Choose an inequality in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ that represents a given real-	Represent real-word problems using equations in the form $px + q = r$ or $p(x + q) = r$. Fluently solve equations in the form $px + q = r$ or $p(x + q) = r$. Solve real-world problems leading to inequalities in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ Graph the solution set for an	Solve real-world problems leading to equations in the form $px + q = r$ or $p(x + q) = r$ both algebraically and arithmetically and identify the similarities and differences between both approaches. Solve real-world l problems leading to inequalities in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ where p, q, and r are specific rational numbers and identify and

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a number line graph that represents a given inequality in the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$.	world problem.	inequality in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ on a number line. Interpret the solution set for an inequality in the form $px + q > r$, $px + q < r$, $px + q \geq r$ or $px + q \leq r$ in the context of the problem.	justify the operations used in each approach. Interpret the solution set for an inequality and explain the reasonableness of the solution set with respect to the context.

Instructional Focus Statements

Level 3:

In grade 6, (6.EE.B.5) students learned that a solution to an algebraic equation is the value(s) that makes the statement true. Students Also solved real-world and mathematical problems by writing and solving one-step equations (6.EE.B.7). In grade 7, students should develop fluency solving contextual problems that can be modeled by linear equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Positive and negative integers, fractions and decimals should be included as values in the contextual problems. An equation is a statement that shows two equivalent expressions using an equal sign. Solving equations should be approached as a process of reasoning rather than a set of steps to follow. Students should work with problems that can be solved arithmetically but can also generalize an algebraic solution that can be applied to the contextual situation. When students work mentally, they often solve problems using the arithmetic approach. It is important for students to see that the arithmetic approach has the same steps as the algebraic approach. As the problems become more complex, it becomes difficult to keep track of the details, therefore, the more efficient the algebraic approach becomes. Encourage students to demonstrate equations using concrete objects such as algebra tiles or balance scales, as well as with visual representations, including drawing pictures, diagrams, bar models, etc.

In grade 6 (6.EE.B.8), students learned to interpret and write simple inequalities, as well as graph solutions of simple inequalities on number lines. Students should use the foundational process of solving equations to begin solving word problems with inequalities. Compare contextual problems that can be modeled with equations to those where an inequality is needed to find a solution set. Students should understand that similar properties are used in solving both equations and inequalities. Students should also recognize one important new consideration in solving inequalities: multiplying or dividing both sides of an inequality by a negative number reverses the order of the comparison it represents. It is useful to present contexts that allow students to make sense of this. Students should also be able to graph their solution set on a number line and interpret the meaning of their solution set with respect to the context of the problem. Facilitate discussion regarding whether all the answers in the solution set make sense in the context of the problem.

Level 4:

Students should extend their understanding of solving contextual problems to recognize whether the problem should be represented with an equation or an inequality and provide justification for their reasoning. Also, students should be able to interpret the solution or solution set and determine its reasonableness to the contextual situation. As students solve equations and inequalities, they should be able to justify their solution or solution set by substituting values in the equation or inequality to determine if the equation or inequality is true. Justification for this should be in written and verbal form using precise mathematical vocabulary.

Geometry (G)

Standard 7.G.A.1 Cluster Heading: A. Draw, construct, and describe geometrical figures and describe the relationships between them.
Solve problems involving scale drawings of congruent and similar geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure a real-world object using a ruler or tape measure. Measure a drawing using a ruler. Find the area of a given triangle or quadrilateral.	Accurately identify figures as congruent or similar. For similar figures, understand that scale factors that are between 0 and 1 result in a reduction in size and scale factors greater than 1 result in an enlargement. Understand that the scale factor for congruent figures is 1. Given a geometric figure and a scale factor, choose the scaled figure. Given a scaled geometric figure and the scale factor that generated it, choose the original figure.	Draw a scale drawing of a geometric figure given a scale factor and the original figure. Given a scale drawing, compute the lengths and area of the original geometric figure. Solve problems involving scale drawings of congruent and similar geometric figures. Reproduce a scale drawing at a different scale. Describe the relationships between the original geometric figure and its scale drawing.	Explain the process of generating scaled drawings using precise mathematical vocabulary. Create and solve multi-step real-world problems involving scale drawings, explaining their reasoning for the chosen solution path.

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Instructional Focus Statements

Level 3:

The concept of scaled drawings is new to students in grade 7. However, students can connect to prior understandings of shapes and their attributes and apply them to scale drawings. Connecting students to real-world examples, such as enlarging photos or reading road maps can help students understand the many uses of scaling. Students are also introduced to congruent and similar figures in grade 7 in this standard cluster. Therefore, students should have the opportunity to explore what makes figures congruent or similar and connect those concepts to scaling.

Students should explore what happens to shapes when a scale factor is applied to a figure. As students multiply the scale factor by each length, they should discover that when the scale factor is between 0 and 1 (non-inclusive), the scaled figure will have the same shape but result in a reduction. When the scale factor is greater than 1, the scaled figure will have the same shape but result in an enlargement. Encourage students to explore what measurement allows the figure to maintain its shape (the angle measures) and why. This discussion will lead to an understanding of similar figures. Students should also be challenged to consider the result if the scale factor applied is equal to one, which leads to an understanding of congruent figures.

Once students have solidified the effects of scale factors on scale drawings, they can apply their understanding of proportional relationships from 7.RP.A.2 and 7.RP.A.3 to discover that the side lengths of similar figures are proportional and solve problems involving scale drawings. For example, given the original figure and the scale drawing, students can find the scale factor and/or any missing side lengths in either figure. This can be particularly challenging for students when given the scale drawing and the scale factor that produced it and asked to find the lengths of the original figure. Connecting students back to the effect the scale factor had on the original figure to produce the scale drawing is essential to developing the understanding that if the scale factor resulted in a smaller figure, then the original figure must be larger and vice versa.

In grade 6, students found the area of triangles, quadrilaterals, and polygons by decomposing the figures into rectangles and triangles as needed. In this standard, students will apply their understanding of area to compare areas of both the original and the scaled drawings. For example, they should be allowed to discover that the areas of similar polygons change at a different rate than the lengths of the sides, Students will not be expected to know the area changes by the scale factor squared, since squaring is not solidified until grade 8 (8.EE.A.2). This relationship will be explored further in high school. Therefore, the focus in grade 7 is on developing the understanding that the area ratio will be different to avoid the misconception that the scale factor should simply be applied to either measurement. Instead, students should be provided with problems that require them to calculate actual measures after they have generated a scale drawing such as area and perimeter from scale drawings using both whole number and fractional measurements.

Level 4:

Challenge students to explain using precise mathematical language the effects of the scale factor on the figure connecting it to the process of generating a scale drawing and justify why applying another scale factor on the scale drawing results in another unique scale drawing. Students at this level should be challenged to create and solve their own multi-step real-world problem that involves scale drawings and explain their reasoning for the solution path they chose using precise mathematical language.

Students can further explore scale drawings of congruent figures as well as scale drawings of similar figures. Given two congruent figures, A and B, a scale drawing of figure A would also be the scale drawing of figure B with the same scale factor. However, given two similar figures, C and D, a scale drawing of figure C would require a different scale factor to be the result of figure D. It is essential that students understand that when you multiply the dimensions of one scale drawing by a scale factor, you produce another unique scale drawing. They can develop this understanding through their own exploration.

Geometry (G)

Standard 7.G.A.2 Cluster Heading: A. Draw, construct, and describe geometrical figures and describe the relationships between them.
Draw triangles with given conditions: three angle measures or three side measures. Notice when the conditions determine a unique triangle, more than one triangle, or no triangle.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize examples of geometric shapes with given conditions.	Choose a geometric shape that represents a given set of conditions.	Construct triangles given three angle measures or three side lengths. Recognize when a given set of line segments will form a triangle. Recognize that the sum of the angles in a triangle is always 180°. Recognize when conditions form a unique triangle, more than one triangle, or no triangle.	Create a set of conditions that form a unique triangle, more than one triangle, or no triangle and explain the reasoning using precise mathematical language.

Instructional Focus Statements

Level 3:

In grade 5 (5.G.B.3), students study the sides and angles of two-dimensional figures including triangles in order to classify them. In standard 7.G.A.2, students will explore sides and angles in triangles and draw conclusions about what makes a triangle. The focus is on sketching geometric shapes with given conditions which is foundational for understanding congruence and similarity in grade 8 and high school. Students construct triangles from three given sides or three given angles with a variety of tools, including by hand using rulers, protractors, and by using technology. Students will learn the

concepts behind the Triangle Sum Theorem and the Triangle Inequality Theorem but are not expected to know these theorems by name. These theorems will be solidified for students in grade 8 (8.G.A.2).

To practice geometric deduction and build conceptual understanding, students should discover with visual representations or models that the sum of the angles in a triangle is 180°. Students can measure the angles of a triangle using a protractor or technology and adding them together and/or by placing all three angles from a triangle together to visually see they are supplementary and form a straight angle. These activities should be applied to multiple triangle types, so that students can draw the conclusion that regardless of whether it is an acute, right, or obtuse triangle the sum of angles will always be 180°.

Likewise, by providing students with manipulatives that consist of segments of varying lengths and having them attempt to form triangles using any combination of three segments, students can discover that not all combinations of three segments can form a triangle. Challenge students to find multiple combinations that do not form a triangle and hypothesize why. This should lead to the conclusion that the sum of two side lengths of a triangle must always be greater than (not equal to or less than) the third side.

Students should be able to determine when given three measures if these conditions result in one, more than one, or no triangle. For example, when given three side lengths that will form a triangle, students should recognize that this results in a unique triangle. This concept leads to an understanding of congruence that will be formalized in high school (G.CO). Three side lengths that do not meet those requirements result in no triangle. If given three angle measures that add up to 180°, students should recognize that not only will they form a triangle, but multiple triangles exist that have the same three angle measures. However, they can vary in size depending on the length of the sides. This results in more than one possible triangle, which leads to an understanding of similar triangles, which will be formalized in grade 8 (8.G.A.2). Three angle measures that do not add up to 180° result in no triangle.

Work with the Triangle Inequality Theorem can be nicely integrated as students work with solving simple inequalities. Additionally, by sketching geometric shapes that adhere to a given set of conditions, students lay the foundation for the concepts of congruence and similarity and for the practice of geometric deduction that will grow in importance throughout the high school course work.

Level 4:

As students deepen their understanding, they should be able to make generalizations about constructing triangles given a set of conditions. In doing so, students should be able to not only identify that a given set of conditions is one, more than one, or no triangle, but also create examples to support their generalizations and explain their reasoning using precise mathematical language.

Geometry (G)

Standard 7.G.B.3 Cluster Heading: B. Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.
Know the formulas for the area and circumference of a circle and use them to solve problems. Explore the relationship between the radius, the circumference, the area of a circle, and the number π.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the parts of a circle that represent the radius, diameter, area, and circumference. Relate the circumference as a perimeter measurement of the distance around a circle. Recognize that pi has an approximate value of 3.14.	Understand that all the points on a circle are the same distance from the center. Find the diameter given the radius and radius given the diameter. Identify the formula used to find area of a circle. Identify the formula used to find circumference of a circle. Informally explain the relationship between circumference and diameter.	Know the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. Determine the radius and/or the diameter of a circle when given the circumference. Determine the area of a circle when given the circumference. Express the area and circumference of a circle in terms of pi and as an approximate numerical value. Informally explain the relationship between the circumference and area of a circle.	Justify the relationship that exists between circumference and area written in mathematical explanations. Create and solve a real-world problem which involves calculating the area and/or circumference of a circle or finding the radius or diameter given the circumference.

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Instructional Focus Statements

Level 3:

Students have worked with the concepts of area and perimeter since grade 3. Prior to grade 7, this experience was limited to polygons. Students now build on that understanding to solve problems involving the area and circumference of circles. Therefore, grade 7 is students' first formal exposure to the value of π (pi) and its application to circles. It is also students' first formal exposure to the parts of a circle including the radius, diameter, and circumference, so instruction should include the introduction to this vocabulary and what they represent. For example, allow students to experiment with either a compass or a string to draw a circle and explain why these tools work. This experiment will help students become familiar with the idea that all the points on a circle are the same distance from the center and that distance is the radius. This also leads to an informal definition of a circle.

Because students are often confused by the formulas for the area and circumference of a circle, instruction should focus on helping students discover the relationship between the two. Prior to providing the formula, students should be given ample opportunity to discover the mathematical relationship that exists between the circumference of a circle and its diameter as well as the mathematical relationship that exists between the area of a circle and its radius. Activities that allow students to repeatedly measure and compare the circumference and diameter of different-sized circles will lead them to discover that circumference is approximately three times the size of the diameter, which in turn leads to the derivation of pi. Students should also be able to explain the relationship between the two formulas for circumference: $C = \pi d$ and $C = 2\pi r$ and fluently move between them depending on what information is given in the problem.

It is important that students are exposed to activities that allow them to apply the formulas to solve a wide variety of mathematical and real-world problems. This means that in addition to being asked to find the area given the radius, they should be presented with problems that provide the diameter. Likewise, students should be asked to find circumference given the diameter or the radius. They should also be given problems that require finding the radius or diameter given the circumference. However, it is not appropriate in grade 7 for students to find the radius given the area since it requires taking a square root which is not addressed until grade 8 (8.EE.A.2). Additionally, challenge students to use their geometric reasoning by including problems that ask for area or circumference for whole circles, as well as quarter circles and semi-circles. Additionally, it is important that students understand exact answers versus approximate answers when rounding the value of pi to 3.14, recognizing that even a calculator must round π. Students should be expected to give solutions in terms of π and as approximate decimal values.

Students should explore the relationship between the circumference and area of a circle. As students begin to move fluently between finding area or circumference given the radius or diameter and finding the radius or diameter given the circumference, they can reason through finding the area given the circumference. Discussion should again include real-world examples of when exact answers in terms of π would be more valuable than approximate decimal answers and when approximate answers are acceptable.

Level 4:

Students at this level should be able to demonstrate conceptual understanding of the relationship between the circumference and area of a circle. By building on the development of their prior knowledge of area and perimeter, students should be able to informally derive the formulas for the area and

circumference of a circle. One way to model this is by slicing a circle into thin pie-shaped pieces and arranging the pieces to form an approximate parallelogram shape. Students should use their prior knowledge of how to find the area of a rectangle or parallelogram, relating it to the parts of the circle (before it was transformed). The teacher can facilitate a discussion to lead students to the formulas for circumference and area of a circle. Students should recognize that the height of the parallelogram is the radius of the circle or r, and that the base of the parallelogram is equal to one-half of the circumference of the circle or ½(2πr), therefore, the area of the parallelogram is r ½ (2πr) = πr^2. Using precise mathematical vocabulary, students can use this activity to justify the relationship between circumference and area of a circle and how that relationship is used to derive the formulas for each.

Geometry (G)

Standard 7.G.B.4 Cluster Heading: B. Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.
Know and use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Write and solve simple equations. Understand that an angle is measured by the degree it opens. Recognize when angles are adjacent versus non-adjacent. Recognize a right angle and a straight angle and know their measurements are 90° and 180° respectively.	Classify pairs of angles as supplementary, complementary, vertical, and adjacent angles in a diagram. Recognize that supplementary angles have a sum of 180° and complementary angles have a sum of 90°. Recognize that vertical angles have the same measure.	Apply the properties of supplementary or complementary angles to write and solve an equation to find unknown angle measures. Apply the relationship of vertical angles to write and solve an equation to find unknown angle measures. Apply the properties of adjacent angles to write and solve an equation to find unknown angle measures. Justify the equations and the solution paths to multi-step problems involving angle relationships using precise mathematical language.	Create problems related to finding unknown angle measures that involve supplementary, complementary, vertical, and adjacent angles in a real-world setting.

TN Department of Education

Instructional Focus Statements

Level 3:

In grade 4, students drew angles and classified them as right, acute, obtuse, straight, or reflex (4.G.A.1). In grade 7, students apply their understanding of angles to explore relationships of pairs of angles.

Students should be provided opportunities to explore supplementary, complementary, vertical, and adjacent angles and their relationships to one another through measuring using a protractor and/or technology and identifying patterns to discover their properties. For example, when given a picture with two non-perpendicular intersecting lines, students should measure all the angles with a protractor to discover that some pairs have a sum of 180° (supplementary) and other pairs have the same measure (vertical) connecting those discoveries to the appropriate terms. Students should also work with examples of supplementary and complementary angles that are not adjacent, so they learn that these terms apply to any angles that add up to 180° or 90° respectively.

Students should be expected to use these angle relationships to write and solve multi-step problems by applying what they know about the properties of these different types of angle pairs. Discussion should lead students to find angle relationships from a context and/or a diagram and justify their equations by sharing their reasoning. While students can initially verify the measure of an unknown angle with a protractor, it is important that students recognize that drawings are not always drawn to scale. Therefore, they must rely on given measurements, either numerical or expressions, to find a missing angle measure or value. Students will apply their learning from standard 7.EE.B.4 on writing and solving equations to the context of the relationship of the angles.

This standard provides an opportunity for students to check for the reasonableness of their solutions. Instruction should guide students to connect the geometry to the equation and their solution path. They should recognize that the equation should explicitly reveal the property of the pair of angles in the problem. For example, if they are looking at complementary angles, their equation should have the numbers or expressions representing the two angles added together to equal 90°. Also, since angle measures cannot be negative, the resulting angle measure should also not be negative. A strong understanding of angle relationships will be essential as students later use informal arguments to establish facts about angles in grade 8 (8.G.A.2) and in high school.

Students may have been introduced to naming and notation of angles in grade 4, but students in grade 7 are expected to attend to precision (MP6) and should be able to name angles appropriately to accurately call out these angle pairs and recognize angles that are referenced by name using correct notation. Therefore, instruction should include proper notation for naming angles including by the vertex, by three points, and by number labels (i.e., ∠B, ∠ABC, ∠CBA, ∠1).

Level 4:

As students strengthen their understanding, they should apply their learning from simple diagrams to real-world contexts. Students should be able to analyze a complex situation and make connections between geometric concepts and algebraic thinking to write and solve multi-step equations that represent the angles formed to solve for the measurement of missing angles within the context. As this learning is solidified, students should create their own real-world problems related to finding unknown angle measures using the relationships between pairs of angles. Students should be able to justify the connections between their created situation and their solution paths of these complex situations using precise mathematical language.

Geometry (G)

Standard 7.G.B.5 Cluster Heading: B. Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.
Solve real-world and mathematical problems involving area of two-dimensional figures composed of triangles, quadrilaterals, and polygons, and volume and surface area of three-dimensional objects composed of cubes and right prisms.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the area of triangles in real-world and mathematical problems. Find the area of quadrilaterals in real-world and mathematical problems.	Find the surface area of rectangular prisms, including cubes, in real-world and mathematical problems. Find the volume of rectangular prisms, including cubes, in real-world and mathematical problems. Find the area of two-dimensional polygons that can be decomposed into rectangles and/or triangles.	Solve real-world and mathematical problems involving area of two-dimensional figures composed of triangles, quadrilaterals, and polygons. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of cubes and right prisms.	Use reasoning to find possible dimensions when given the surface area or volume of a right prism. Use precise mathematical language to explain the relationships between area, volume, and surface area. Justify the relationship between the volume of a pyramid and a prism.

Instructional Focus Statements

Level 3:

In grade 6 (6.G.A.1) students composed and decomposed various polygons to find area. Students in grade 7 should explore different and more complicated figures that can be decomposed into triangles, quadrilaterals, and polygons to find their area. Students should work with manipulatives and drawings to gain an understanding that familiar shapes can be put together to create composite shapes, and unfamiliar shapes can be decomposed into non-overlapping, adjacent familiar shapes. Students should use this knowledge to build an understanding that the area of composite shapes is equal to the sum of the areas of the non-overlapping, adjacent shapes. By allowing students to use manipulatives and drawings to find the area of two-dimensional figures, they will deepen their conceptual understanding of area and avoid the confusion that often occurs when distinguishing between area and perimeter.

In grade 6 (6.G.A.4), students found surface area of three-dimensional objects using nets made up of rectangles and triangles. Students in grade 7 should build on this learning by exploring nets of more complex three-dimensional solids. For example, what would the net of a prism or pyramid with a trapezoid base or a pentagon look like and how can it be used to find the surface area? Instruction should be focused on the multi-step process of decomposing the unfamiliar shapes into familiar shapes.

Students began finding volume in grade 5 (5.MD.C.5) by filling a rectangular prism with unit cubes. In grade 6 (6.G.A.2), they connected that concept to two formulas used to find the volume of a rectangular prism. To build on this prior knowledge of volume, instruction in grade 7 should continue to focus on the idea that volume is the space an object holds. This is reinforced by again having students fill a prism with unit prisms. For example, fill a rectangular prism with unit cubes one layer at a time until it is full. They can connect this idea of layers to repeated addition and thus multiplication, then generalize that volume of a prism can be calculated by the area of the base, regardless of the shape of the base, multiplied by its height. Through exploring the volume of prisms with differently shaped bases, such as a triangular prism, students should discover that the formula $V=lwh$ can only apply to a rectangular prism, while $V=Bh$ applies to any prism.

Students should also explore finding the volume of pyramids by comparing the volume of a pyramid with the volume of a prism. For example, provide a right prism and pyramid that have congruent bases and the same height and fill the pyramid with water or sand. Then have students predict how many times the contents of the pyramid can be poured into the prism until it's full. This will help students see that the prism will hold three times the volume of the pyramid and thus, the volume of the pyramid is one-third the volume of the prism (Prism: $V = Bh$; Pyramid: $V = \frac{1}{3}Bh$).

It is important for students to understand that an uppercase B represents something different than a lowercase b in geometry formulas. Specifically, B represents the area of the base, while b represents an edge length. Students often struggle with recognizing this difference. However, allowing students to develop a conceptual understanding of volume by building models of solids and filling them with unit cubes or prisms one layer at a time will help them connect to the formula. By reinforcing this conceptual understanding of what volume is, students also better understand why volume is measured in cubic units.

The focus should continue to be on developing a conceptual understanding of area, surface area, and volume rather than simply rote memorization of formulas. Students need to continue exploring the concepts of area, surface area, and volume with concrete materials where they are encouraged to measure to find area, fill to find volume, and use nets to determine surface area. Students should be expected to apply this knowledge to solve problems in real-world contexts. Students often confuse when to use area, surface area, or volume in real-world problems. Exploring these concepts with concrete materials rather than rote memorization of formulas can help students better distinguish when they need to use each measurement.

Level 4:

Students at this level should be expected to find the area, volume, and surface area of complex polygons that are presented in a visual mathematical or real-world context. After students have developed a deep understanding of area and have had the opportunity to develop their own strategies, they can explore additional strategies, including using the apothem to find the area of a regular polygon. Use of the apothem to find area is an optional strategy and not an expectation of this standard. Students will further develop this strategy and others in high school.

Students can also be challenged to apply their knowledge of area, volume, and surface area to reason through finding possible dimensions for two and three-dimensional shapes when given the area, surface area, or volume. For example, students could then be asked for possible dimensions of a right triangular prism with a volume of 480 square inches and compare their results.

Instruction should include opportunities for discourse where students are given complex three-dimensional shapes that involve various combinations of rectangular and triangular prisms together. Students should be expected to justify the methods or formulas they used to find volume and surface area and connect them to parts of the complex object. Likewise, they could generate context for a two- or three-dimensional polygon for others to analyze and solve. Discourse should be facilitated so that students connect the context to the object and provide explanations to support the use of each method or formula.

Students should be expected to use precise mathematical language to justify the relationships that exist between area, volume, and surface area.

Statistics and Probability (SP)

Standard 7.SP.A.1 Cluster Heading: A. Use random sampling to draw inferences about a population.

Explore how statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define random sampling. Collect sample data from a population. Generate a statistical question in a real-world context.	Explain the importance of random sampling in statistics. Differentiate between a sample and population. Recognize invalid sampling techniques.	Determine if a sample is representative of a population. Determine if a sample is biased. Identify factors that would contribute to bias. Generate random samples that are representative of a larger population.	Conduct a survey and explain why the method used will produce a sample that is unbiased and representative of the population. Explain adjustments needed to produce a representative sample when given an invalid sampling technique, Critique examples of random sampling as statistical tools using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6, students formulated statistical questions and collected data to answer those questions. In grade 7, students improve the production of data and understand the importance of selecting random samples. This standard introduces students to random sampling and how those samples can be used to gather information about the populations from which they are drawn. Facilitated discussion should lead students to an understanding of statistics, why it is studied, and how it is useful in real-world applications. Opportunities to collect relevant data from a large population should reveal the difficulty in

gathering statistics on an entire population. To emphasize this point, students learn that sampling is the process of taking a subset of subjects that is representative of the population and collect data on that subset. An example of a random sample is taking a list of names at a school and selecting every fourth person to be in the sample to represent the population of the school. Students should be exposed to statistical questions that would require random sampling and could not be easily answered using other sources, such as the Internet.

Providing opportunities for students to analyze statistical data while considering bias and whether the sample is representative of the population is essential. In order to show the benefits of randomization, students should be presented with scenarios that include random and non-random samples and discuss how each might affect the validity of the results drawn from the samples. While many students will understand the need to produce a random sample, exposure to situations that are random but not representative of the population, will provide opportunities for discourse using precise mathematical vocabulary around the use of both conditions. In addition, instruction should include discussions around sample sizes so that students understand how samples that are too small can skew results.

Level 4:

At this level, students should be challenged to go beyond generating random samples, to explaining the statistical process used to generate their samples, using precise mathematical vocabulary. Instruction should include opportunities for students to justify their own sampling and explain why the sample is representative of the population. When given an invalid sampling technique, students at this level should be able to explain why it will not produce a representative sample and explain any adjustments that need to be made to the process such as increasing the size of the sample or eliminating opportunities for bias.

Statistics and Probability (SP)

Standard 7.SP.A.2 Cluster Heading: A. Use random sampling to draw inferences about a population.

Collect and use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the benefits of random sampling. Differentiate between a sample and population. Determine if a sample is representative of a population.	Generate multiple samples that are representative of a larger population. Describe a sample using a measure of center or variation. Differentiate between sample measures and population measures.	Recognize that increasing sample size affects sampling variability. Make inferences and generalizations about a population by collecting and using data from random samples. Justify inferences and generalizations using appropriate mathematical vocabulary. Recognize how sample variations affect the accuracy of predictions.	Explain the impact of sample size on sampling variability with precise mathematical language. Assess the accuracy of a prediction using multiple random samples.

Instructional Focus Statements

Level 3:

In grade 6, students formulated and answered statistical questions. In grade 7, students are introduced to random sampling and learn how those samples can be used to gather information about the populations from which they are drawn (standard 7.SP.A.1). Instruction should include multiple opportunities

to collect data samples and make generalizations about populations. This can be done using actual experimentation or simulation methods. Discourse should focus on the determination of the sample mean or proportion, and students should be encouraged to think about how different samples would result in different values. For example, collect two random samples of 100 students about their school lunch preferences and use the samples to predict the most reasonable estimate for students who prefer pizza for lunch. Students should use their understanding of proportional relationships to determine reasonable estimates.

Students should engage in experiments and simulations that illustrate how different samples from the same population can differ to solidify understanding of sampling variability. For example, a sample of 25 survey responses might yield a different proportion than a second sample of 25 survey responses. A misconception is that only one of the samples is correct. Through collecting multiple data collections, students should realize that none of the samples are exactly the population, but together they provide a good picture of the population. In addition to noticing variations in samples, students should be expected to explain variations in samples. As students collect and use multiple samples of data to make generalizations about a population, issues of variation in the samples will provide opportunities for discussion and may lead to further investigations.

Activities that allow students to compare sample measures to population measures are essential for students to recognize and understand that the two are not the same. Engagement opportunities should reveal that conclusions drawn from random samples generalize beyond the sample to the population from which the sample was drawn; however, these are only estimates and there will be some discrepancy between the two. Combining sample data on a dot plot will provide a visual illustration depicting how the sample measures vary. Questions should be posed so that students notice variation in estimates and can estimate about how far off the prediction about the population could be. This experience in collecting and analyzing univariate data will be beneficial as students go on to collect and represent bivariate data in grade 8.

Level 4:

Students at this level should be challenged and expected to move beyond recognizing sample variability on a dot plot by explaining variability between sample measures using precise mathematical language. Students should justify placement of dots and lead others to see how close the sample measure is to the population measure. When presented with sample measures, students should be challenged to explain how sample size could affect variability. When posed with objections, students should be encouraged to use multiple sample sizes to show how a change in sample size results in a change in variability. When given a prediction about the winner of a school election, students at this level should be expected to assess the prediction based on sample data. Students should be challenged to use the variation in sample data to explain how the prediction is, or is not, justified.

Statistics and Probability (SP)

Standard 7.SP.B.3 Cluster Heading: B. Draw informal comparative inferences about two populations.

Informally compare the measures of center (mean, median, mode) of two numerical data distributions with similar variabilities. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team; on a dot plot or box plot, the separation between the two distributions of heights is noticeable.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize when an overlap exists between two data sets. Determine the center of numerical data sets.	Represent real-world data sets using dot plots, box plots or histograms. Solve problems involving data from a dot plot, box plot, or histogram.	Compare the measures of center (mean, median, mode) of two data sets with similar variability. Model and compare two real-world data sets by measuring the difference between their measures of center.	Determine and explain when the difference in the measures of center of two data sets is meaningful.

Instructional Focus Statements

Level 3:

In grade 6, students displayed single data sets in various ways and described data using measures of center and variation. In grade 7, students build on this learning to compare data sets of two distinct populations. This is the students' first experience with comparing two data sets, so instruction should begin with the analysis of one data set and then add a second for comparison. Discussion should be facilitated so that students consider measures of center (mean, median, mode). Students should recognize that when comparing the means of two data sets, the mean of each sample will most likely not be the same.

It is important that students are exposed to contextual examples, which will allow them to not only calculate measures of center, but also understand their meaning in terms of the context. For example, a student compares the median number of texts per day sent by middle school students to the median number of texts send per day by high school students. The data sets have the same range, but different medians. How might dot plots of the data be

different? How might they be the same? Students will have prior knowledge of graphs such as dot plots, box plots or histograms. Give students opportunities to make observations based on the graphs, prompting them to describe clusters, gaps, centers and variability. Numerical data that is of interest to them (e.g., sports, government, recent events) should be incorporated and provided when possible. When comparing large data sets, calculators can be used to aid in computations so that student discussion is focused on the comparison of the data sets. These understandings will be essential as students go on to model bivariate relationships in grade 8.

Level 4:

In addition to comparing data sets, students at this level should be challenged to determine and explain if the difference in the two data sets is meaningful or great enough to assert a distinction between the data sets. Given two raw data sets, students should be challenged to draw conclusions based on the difference of the means. Encourage students to debate and justify their observations. For example, if there is only a small difference in the mean prices of the two stores, one might conclude that there is no real benefit in shopping at one or the other. Students should be challenged to justify this conclusion using precise mathematical language and appropriate graphs.

Statistics and Probability (SP)

Standard 7.SP.B.4 Cluster Heading: B. Draw informal comparative inferences about two populations.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a 7th grade science book are generally longer than the words in a chapter of a 4th grade science book.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe a sample using measures of center. Describe a sample using measures of variability.	Determine the appropriate measure of center or variability for describing a single data set. Make inferences about a single population using data from random samples. Draw comparative conclusions about two populations using graphs.	Determine the appropriate measure of center or variability for comparing two data sets. Draw valid comparative inferences about two populations using measures of center. Draw valid comparative inferences about two populations using measures of variability. Identify valid inferences related to the comparison of two data sets and justify why they are valid.	Critique the selection of a measure of center or variability that is used to compare two data sets. Critique comparative inferences about two populations using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6, students represented data using graphs and determined measures of center and variability for single data sets. Students now compare two sets of data using appropriate measures of center and variability. This standard also connects to standard 7.SP.A.3 as students are now asked to draw

inferences. An inference is a conclusion reached based on evidence or reasoning. Facilitated discussion should lead students to describe what they notice about the data and wonder about what might be happening in the population. In comparing two data sets, challenge students to look for evidence using measures of center or variation to support their claim.

Instruction should include opportunities to analyze data sets that are presented in multiple formats (e.g., dot plots, box and whisker plots, histograms etc.). In comparing the data sets, students should have opportunities to determine which measures of center (mean, median, mode) or variability (range, interquartile range) should be used for a valid comparison. Students should make informal comparative inferences about the data sets using precise vocabulary. Rather than just determining measures of center and variability, it is important that students use them to support their comparative inferences and engage in discussion with their peers about around the validity of their inferences. An understanding of statistics and comparative inferences is necessary as students go on to compare center and variation of two or more different data sets in high school.

Level 4:

Students at this level should have a thorough understanding of measures of center and variability and how they are used to compare data sets. Students should be challenged to critique the selection of a measure of center or variability that is used by a peer to compare two data sets. In addition, students should be challenged to make adjustments needed to create a more appropriate comparison. Multiple opportunities to critique comparative inferences will be beneficial in building students' ability to make decisions using multiple data sets.

Statistics and Probability (SP)

Standard 7.SP.C.5 Cluster Heading: C. Investigate chance processes and develop, use, and evaluate probability models.

Recognize that the probability of a chance event is a number between 0 and 1 and interpret the likelihood of the event occurring.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define probability as the likelihood of a chance event occurring.	Define the complement of an event as the chance of the event not occurring. Recognize that the probability of a chance event can be represented as a rational number from 0 to 1.	Recognize that an impossible event has a probability of 0 and that a certain event has a probability of 1. Recognize that a probability closer to 0 represents an unlikely chance event, a probability close to $\frac{1}{2}$ represents an equally likely (to occur or not occur) chance event and a probability closer to 1 represents a likely chance event. Understand that a chance event and its complement has a sum of 1. Conduct simple experiments and calculate probabilities as fractions, decimals, and/or percent. Determine where probabilities of simple events lie on the probability scale.	Justify the categorization of events as likely, unlikely or equally likely (to occur or not occur) using precise mathematical vocabulary. Use reasoning to determine and explain where a probability lies on the scale when expressed as a fraction such as $\frac{3}{8}$.

TN Department of Education

Instructional Focus Statements

Level 3:

In grade 6, students used ratio and rate reasoning to solve real-world and mathematical problems. In grade 7, they expand their understanding of ratios and develop an understanding of probability. This is the students' first formal introduction to probability. Students should recognize that the probability of any single event can be expressed in terms such as impossible, unlikely, equally likely, likely, or certain. Discussion should include familiar events in students' lives and the likelihood of those events occurring. Initial discussions should relate the likelihood of these events to probabilities of 0, ½ and 1. Students should engage in discourse around the meaning of numerical probabilities, understanding that the closer the probability is to 1, the greater the probability the event will occur, and the closer the probability is to 0, the less likely the probability the event will occur. A probability close to ½ represents an equally likely to occur or not occur event. Students should understand that the probability of 1 is certain and 0 is impossible.

Since students were exposed to the number line in previous grades, they should have multiple opportunities to place probabilities on the number line, or probability scale. Instruction should progress to include rational number probabilities between 0 and 1 so that students understand that a probability can be expressed as a fraction, decimal or percent. It is important to discuss why probability cannot be expressed as a number greater than 1. Conducting simple experiments (e.g., flipping a coin, tossing dice, spinning a spinner, etc.) will solidify students' understanding of probability and allow them to connect numerical probabilities to verbal descriptions. This initial understanding of probability will be essential as students go on to experiment with relative frequencies.

Level 4:

Students at this level can categorize events but can also justify the categorization of those events using precise mathematical vocabulary and the probability concept. Students should be challenged to determine and explain where a probability lies on the scale when it is presented as a non-benchmark fraction. Students at this level can justify their placement of these probabilities by reasoning about their proximity to benchmark fractions.

Statistics and Probability (SP)

Standard 7.SP.C.6 Cluster Heading: C. Investigate chance processes and develop, use, and evaluate probability models.

Calculate theoretical and experimental probability of simple events.

7.SP.C.6.a Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

7.SP.C.6.b Calculate the theoretical probability of a simple event.

7.SP.C.6.c Compare theoretical probabilities to experimental probabilities; explain any possible sources of discrepancy. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize that the probability of a chance event can be represented as a rational number from 0 to 1. Express the probability of a single event using the appropriate terms impossible, unlikely, equally likely, likely, or certain.	Conduct simple experiments and calculate probabilities as fractions, decimals and/or percents. Recognize theoretical probability as what should mathematically occur and experimental probability as the actual outcome of an experiment. Recognize relative frequency as the observed number of successful events for a finite sample of trials.	Determine the theoretical probability of a simple event. Determine the experimental probability of a simple event. Differentiate between experimental and theoretical probability of a simple event. Estimate the probability of an event by collecting data and observing its long-run frequency. Compare probability models to observed frequencies and explain any discrepancies between the model and observed frequencies.	Explain orally and in writing any discrepancies between theoretical and experimental probabilities using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 7, students are formally exposed to probability for the first time. While students developed an understanding of the meaning of probability with standard 7.SP.C.5, they now learn the difference between theoretical probability and experimental probability. Theoretical probability is what should mathematically occur, and experimental probability is the actual outcome of an experiment. Relative frequency is the observed number of successful outcomes in a set number of trials. Instruction should include multiple opportunities to collect data on hands-on chance events such as spinning a spinner, rolling a die or flipping a coin and making conjectures about the probability of those events. Instruction should also include opportunities to pool data from experiments and observe long-run frequencies in small groups. Discussion should focus on the idea that as the number of trials increase, the experimental probability approaches the theoretical probability. Students should use their understanding of proportional relationships to calculate theoretical probabilities. Students should also realize that discrepancies between theoretical probability and experimental probability do not necessarily indicate an error/inaccuracy. The use of simulation with technology will be beneficial as students learn to make predictions about the relative frequency of events.

Level 4:

In addition to recognizing discrepancies between probability models and observed frequencies, students at this level should be able to explain any discrepancies orally and in writing using precise mathematical vocabulary. Students should have multiple opportunities to perform experiments, comparing the relative frequency to the known theoretical probabilities. In comparing the two, students should engage in discourse around the significance of the discrepancy. Students should be able to explain that small differences in the relative frequency and theoretical probability does not indicate an error. For example, when conducting 20 trials a relative frequency of 9/20 is close to 1/2 and does not indicate an error. In the event there is a true discrepancy, students should be expected to provide reasons for the discrepancy. One possible reason could be that there were not enough trials conducted in the experiment. A student at this level should be able to explain why more trials are needed and conduct those trials to show how the discrepancy could be resolved.

Statistics and Probability (SP)

Standard 7.SP.C.7 Cluster Heading: C. Investigate chance processes and develop, use, and evaluate probability models.

Develop a probability model and use it to find experimental or theoretical probabilities of events.

7.SP.C.7.a Use a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

7.SP.C.7.b Develop a probability model, including non-uniform models, by observing frequencies in data generated from a chance process. Use the model to estimate the probabilities of events. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Conduct simple experiments and express probabilities as fractions, decimals and/or percents. Recognize theoretical probability as what should mathematically occur and experimental probability as the actual outcome of an experiment.	Approximate theoretical probability and experimental probability in real-world situations. Recognize that probability estimates are affected by sample size. Understand the difference between uniform and non-uniform probability models.	Develop appropriate probability models to find probabilities of events with equally likely outcomes. Develop appropriate probability models to find probabilities of events with outcomes that are not equally likely. Develop probability models by observing frequencies and approximating the probability using hands-on experiments.	Design and conduct probability experiments, making conjectures about the outcomes using precise mathematical vocabulary. Justify conjectures made about experimental probabilities.

Instructional Focus Statements

Level 3:

In grade 7, students learn that probabilities are useful for predicting what will happen over the long run. Applying what they learned about theoretical probability, students predict frequencies of outcomes using appropriate probability models. Uniform probability models are those where the likelihood of each outcome is equal. The probabilities of the models can be either theoretical or experimental with outcomes that may or may not be equally likely. For example, the probability of a balanced coin landing on heads or tails is ½, but the probability of a tossed thumbtack landing point up is not necessarily ½ as the two possible outcomes may not be equally likely.

Instruction should include multiple opportunities to conduct experiments so that students recognize appropriate models and understand that probability estimates are affected by sample size. Students should also be provided multiple opportunities to replicate experiments, comparing the results to theoretical probabilities. Experiments can be conducted using physical objects but can also be conducted using random generation devices such as bag pulls, spinners, number cubes, coin toss, and colored chips. Extensive practice with experiments will be beneficial as students go on to represent sample spaces for compound events in grade 8.

Level 4:

Students may also develop models for geometric probability, utilizing their understanding of geometric concepts. For example, given a circle inscribed in a square, determine the probability of choosing a point that is in the circle. In addition to participating in experiments, students should be challenged to design simulations, then summarize the data as experimental probabilities and make conjectures about theoretical probabilities. Additionally, students should have multiple opportunities to compare their predictions to the experimental outcomes, as well as justify and refine their conjectures about theoretical probability using precise mathematical vocabulary.

Statistics and Probability (SP)

Standard 7.SP.D.8 Cluster Heading: D. Summarize and describe numerical data sets.

Summarize a numerical data set in relation to its context.

7.SP.D.8.a. Give quantitative measures of center (median and/or mean) and variability (range and/or interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

7.SP.D.8.b. Relate and understand the choice of measures of center (median and/or mean) and variability (range and/or interquartile range) to the shape of the data distribution and the context in which the data were gathered.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Summarize data sets using measures of center. Create dot plots, histograms, and box plots from numerical data.	Summarize data sets using measures of variability. Interpret dot plots, histograms, and box plots in relation to the context of the statistical question.	Determine the most appropriate measure of center and variability to summarize data derived from real world context. Use graphs to describe patterns in data derived from real world context. Recognize how measures of center and variability affect the shapes of data distributions.	Construct viable arguments to explain the selection of statistical measures, predicting how changes in the data affect those statistical measures.

Instructional Focus Statements

Level 3:

In grade 6, students summarize numerical data sets using measures of center (mean and/or median) and variability (range) while describing patterns with reference to the context in which the data were gathered. Students in grade 6 also related the choice of measures to the shape of the data distribution

and the context in which the data were gathered. In grade 7, students build on their understanding of these measures to not only summarize data sets, but also describe any significant deviations from the overall pattern and expand their choice of measure to include interquartile range. Instruction should include opportunities to summarize numerical data derived from statistical questioning using quantitative measures of center (mean and/or median) and variability (range and /or interquartile range). Students should be able to express what a measure of center or variability means in the context of the problem. For example, the mean represents the average height of all the players on a basketball team. Students should be led to realize that the measure of center chosen to describe a data set will depend upon the shape of the data distribution and context of data collection. Students gained an understanding of the mean in grade 6 but should understand that it can be affected greatly by very low or very high data points. Exposure to data sets with extreme values should lead students to recognize when a difference measure might be more descriptive and appropriate for the given data set.

Additionally, exposure to data sets that are not symmetrically distributed should lead students to recognize when data sets are skewed and better represented with a measure other than the mean. To show variation of data, students should engage in activities involving the interpretation of data represented by graphs such as line plots, histograms, and box and whisker plots. Multiple opportunities to summarize data sets in relation to the context will be essential in solidifying students' ability to select an appropriate measure. Discourse should be facilitated so that students have opportunities to justify the measure of center or variability chosen. A thorough understanding of data distributions and patterns will be beneficial as students go on to investigate bivariate measurement data in grade 8.

Level 4:

Students at this level should have a strong understanding of measures of center and variability and be able to determine why a measure was chosen to summarize numerical data sets. Students also understand that the way data is collected, organized, and displayed influences interpretation and can inform decisions. Instruction should include opportunities to explain how a change in the data set would affect the chosen measure. For example, the removal of an extreme value could prompt students at this level to change the chosen measure of center and justify the change in the chosen measure. Students should also be expected to use line plots, histograms, and other data displays to explain how data can be misleading both intentionally and unintentionally.