Sixth Grade Mathematics
Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the "how" and the "why" of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase "procedural skill and fluency" is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the "why" behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

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Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:
The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:
Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

Ratios and Proportional Relationships (RP)

Standard 6.RP.A.1 Cluster Heading: A. Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Make a distinction between ratios and fractions. *For example, the ratio of wings to beaks in a bird house at the zoo was 2:1, because for every 2 wings there was 1 beak. Another example could be for every vote candidate A received, candidate C received nearly three votes*.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a ratio to represent a given situation.	Interpret a given ratio. Write a ratio to represent a given situation in at least one form.	Use ratio language such as for each, or for every, to describe a ratio between two quantities. Express ratios in various forms including fraction notation, using a colon, using the word "to" or as a verbal expression. Explain how ratios and fractions are different.	Create a context or image and use ratio language to describe a relationship between 2 quantities within the problem.

Instructional Focus Statements

Level 3:

In grade 6 students extend their understanding of fractions as a part to whole comparison to include all ratios. Understanding the concept of a ratio is new learning in sixth grade. Students should learn that a ratio is a multiplicative comparison of two quantities within a given situation. This multiplicative relationship can be within the ratio (described as the rate of change) or between two ratios (described as the scale factor). Ratios can express comparisons of part to whole, for example, the number of girls in the class to the number of students in the class. Ratios can also relate one part to another part, for example the number of girls to the number of boys in the class. Students should be able to determine if the ratio is comparing a part to part, or a part to whole relationship and describe the relationship between the two quantities using ratio language, i.e., for each, or for every. While ratios can be written

using fraction notation, students should be able to make a distinction between ratios and fractions. Students should understand that a fraction is a number that describes a single quantity, whereas a ratio is a comparison of two quantities.

Students should also be exposed to rates, a special kind of ratio that compares quantities with different units. Some of the most common rates are mileage (miles per gallon) and wages (dollars per hour). When given a description of a ratio relationship (in both discrete and continuous quantities), students should examine the description carefully to determine the order of the numbers in the ratio. For example, given a recipe that calls for 10 cups of flour for every 2 cups of sugar, students understand that that ratio of cups of flour to cups of sugar would be 10:2, 10 to 2, or 10/2 or two flags are placed every 3.5 feet on the trail to mark the path would be 2:3.5, 2 to 3.5, or 2/3.5. They use precise ratio language and specify the units (cups, feet) when comparing the quantities. Students can create ratios to compare various quantities they find in a real-world setting and use ratio language to describe their findings.

Level 4:

Students should extend their understanding of ratios to creating contextual problems to describe ratios in the real-world. Students should be able to fluently read and write ratios interchangeably in any format as well as be able to identify if each quantity is representing a part or a whole in the context. A student that has mastered this skill should be able to perform an error analysis when given ratios that have been written incorrectly and provide justification for the misconception(s).

Ratios and Proportional Relationships (RP)

Standard 6.RP.A.2 Cluster Heading: A. Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a unit rate *a/b* associated with a ratio *a : b* with *b ≠ 0*. Use rate language in the context of a ratio relationship.

For example, this recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar. Also, we paid $75 for 15 hamburgers, which is a rate of $5 per hamburger. (Expectations for unit rates in 6th grade are limited to non-complex fractions).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a ratio as a unit rate when it is expressed in the form *a* : 1 or 1 : *a*.	Simplify a given ratio in a simple context to a unit rate when the value of *a* in the resultant unit ratio (*a* : 1 or 1 : *a*) is a whole number.	Give a unit rate to represent a ratio embedded in a context. When given a context, use rate language to describe a ratio relationship.	Use rate language to express a unit rate in a non-routine context.

Instructional Focus Statements

Level 3:

Students should build on prior knowledge of ratios (6.RP.A.1) and equivalent fractions (4.NF.A.1) to conceptually understand rate and unit rate. A rate is a ratio that compares two quantities with different units of measure. A unit rate is a special ratio relationship between two quantities where for every x units of one quantity there is 1 unit of another quantity, for example, sixty miles per one hour. Building on 6.RP.A.1, students should be able to distinguish and articulate between the concepts of ratios, rates, and unit rates. Students should calculate unit rates from ratios by apply their previous understandings of equivalent fractions. Note that in sixth grade, expectations for unit rates are limited to non-complex fractions, meaning that both the numerator and denominator of the original ratio will be whole numbers.

Students should explore real-world examples that can be expressed as a part-to-one ratio using ratio language such as *per* and *each* to compare different units or measures. Additionally, students should model the mathematical process of converting between rates in fraction form to word form. Determining, interpreting, and modeling unit rate understanding will lead into future course work with proportionality and linear functions.

Level 4:

Students should extend their understanding of ratios to unit rates in contextual problems. Students should be able to distinguish between the concepts of ratios, rates, and unit rates and explain their reasoning. Additionally, students should, interchangeably, understand and explain a real-world situation in ratio form and write the unit rate that describes the situation using precise/appropriate rate language with words and symbols to compare different units of measure.

Ratios and Proportional Relationships (RP)

Standard 6.RP.A.3 Cluster Heading: A. Understand ratio concepts and use ratio reasoning to solve problems.
Use ratio and rate reasoning to solve real-world and mathematical problems (*e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations*).
6.RP.A.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. *For example, if a runner ran 10 miles in 90 minutes, running at that speed, how long will it take him to run 6 miles? How fast is he running in miles per hour?*
6.RP.A.3c Find a percent of a quantity as a rate per 100 (*e.g., 30% of a quantity means 30/100 times the quantity*); solve problems involving finding the whole, given a part and the percent.
6.RP.A.3d Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Fill in missing values in a table of equivalent ratios. Determine equivalent ratios from a ratio table. Plot points on a coordinate plane when given the x and y values. When given a contextual situation, determine the ratio. For example, 40 dollars for 10 hours of work is a ratio of 40:10.	Use a table and a graph to reason about equivalent ratios. Given a table, represent equivalent ratios on a coordinate plane. Explain a percent as a rate per hundred. When given a contextual situation, determine the ratio using a table. For example, 40 dollars for 10 hours	Solve real-world problems using ratio and rate reasoning using tables, tape diagrams, double number line diagrams, or equations. From context generate tables of equivalent ratios, find a missing number in the table and use the table to plot the ratios on a coordinate graph. Use a table and graph to compare ratios.	Create real-world contextual problems that involve ratio and rate reasoning. Perform an error analysis of ratio and/or rate problems and be able use appropriate mathematical vocabulary to justify your response. Represent ratios by creating tables, tape diagrams, double number line diagrams, and equations.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	of work is a ratio of 40:10, so the unit ratio is 4:1, or four dollars for one hour of work. Convert larger measurement units to smaller measurement units using the unit rate and multiplication.	Solve unit rate problems, including those about unit pricing and constant speed. Use ratio reasoning to convert measurement units within the same system. Solve percent problems involving both finding a percent of a quantity and finding the whole.	

Instructional Focus Statements

Level 3:

Standard 6.RP.A.1 is a prerequisite to this standard. Students must first understand the concept of a ratio before finding equivalent ones. Students will expand their understanding of multiplication and division from earlier grades to solve a variety of ratio and rate problems. This cluster of standards calls for students to understand ratio concepts and use ratio reasoning to solve problems. It is essential that students are given the opportunity to build conceptual understanding by reasoning about ratios, rates, unit rates and percents, using representations such as ratio tables, tape diagrams and double number lines. Understanding ratio and rate concepts will create a foundation for students to recognize and represent proportional relationships in 7th grade (7.RP.A.2).

In grade 6, students should create ratio tables observing the multiplicative relationships between quantities and noticing that the values of equivalent ratios are equal. Students learn to use double number lines in addition to using ratio tables to model the relationship between two quantities. Students connect ratios and fractions and explore ratios and rates used in ratio tables and graphs to solve problems. Students use unit rates, ratios, and multiplicative reasoning (scaling) to create and fill in missing values in a ratio table. Using tables, students should see equivalent ratios as coordinates that can be plotted on a coordinate plane. They should notice that equivalent ratios, when plotted, form a straight line and use this pattern to solve problems.

Students should investigate unit rate problems, including unit pricing and should use reasoning to compare unit prices and determine the better buy. Students should apply these concepts and skills to convert measurement units within the same system of measure (both customary and metric). Initial

quantities used to form ratios should be whole numbers, but the values formed in the process of solving the problem may be fractions or decimals. Students should understand that a percent is a specific type of ratio where the second quantity is 100. They should be challenged to find the percent of a number using multiplicative reasoning and representations such as bar models,100-grids, ratio tables and double number lines to find the whole when a part and a percent are known (*e.g., What is 30% of 60? 75% of what number is 90*?). They should also interpret contextual situations involving percent, such as, "*A soccer player scored 12 goals during this season. This player scored 30% of the shots attempted. How many shots were attempted?"* or *"Only 7 of the 15 dogs at training class were certified as service dogs. What percent of dogs became service dogs?"* Finding percentages when given the numerical quantities, a number and a percent of that number or the number and then determining additional percentage added or subtracted from that number should be given attention.

In grade 5, students worked to convert measurement quantities from a larger unit to a smaller unit using multiplication of the unit rate. Instruction should build on this understanding to now convert from a smaller to a larger by using division of a unit rate. NOTE: Students did not call it a unit rate in grade 5 but the connection should be made in order to help solidify the concept of ratio. In grade 5, students also learned to plot points on a coordinate plane. They will now connect that knowledge to the rate tables they create and interpret to plot the ratios. Students should discover, through exploration of the coordinate plane and class discussion, that when coordinates form a straight line they represent equivalent ratios.

Level 4:

As students solidify their understanding of ratio and rate reasoning they should be challenged to identify and explain the connections between the rate, unit rate, or ratio in various representations such as tables, tape diagrams, double number lines and equations. Students should also be challenged to determine efficient solution paths when solving ratio and rate problems involving percentages and justify their why they have chosen the selected solution path. When students are looking for the unit rate, they should be able to find the unit rate in multiple representations. Students should also have opportunities to create and interpret contextual problems using ratios and rate reasoning in multiple contexts such as miles per gallon, unit cost, and percentages, using precise mathematical vocabulary.

The Number System (NS)

Standard 6.NS.A.1 Cluster Heading: A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Interpret and compute quotients of fractions and solve real-world and mathematical problems involving division of fractions by fractions (e.g., connecting visual fraction models and equations to represent the problem is suggested).

For example, create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ *because* $\frac{3}{4}$ *times* $\frac{8}{9}$ *is* $\frac{2}{3}$ $((\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc})$ *Further example: How much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How wide is a rectangular strip of land with length of* $\frac{3}{4}$ *mi and area of* $\frac{1}{2}$ *square mi?*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Compute the quotient of a fraction divided by a unit fraction with the same denominator when a visual fraction model is provided.	Compute the quotient of a fraction divided by a unit fraction with the same denominator. Compute the quotient of a fraction divided by a fraction resulting in a quotient that is a whole number when a visual fraction model is provided.	Compute the quotient of a fraction divided by a fraction. Solve contextual problems involving partitive and quotative division of fractions by fractions. Interpret solutions to real-world problems involving partitive and quotative division of fractions by fractions. Model both partitive and quotative division using visual fraction models.	Solve real-world problems involving division of fractions by fractions and interpret solutions in complex contextual problems. Create real-world problems involving division of fractions by fractions and interpret the solution using precise mathematical language. Explain the connection between computing quotients of fractions by fractions and visual fraction models and equations that represent the problem.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			Model both partitive and quotative division using visual fraction models and explain each both in verbal and written form using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6, students continue the work with operations on fractions by computing quotients of fractions involving division of fractions by fractions. In grade 5 (5.NF.B.7), students began finding quotients of fractions by dividing a unit fraction by a whole number and dividing a whole number by a fraction with an instructional focus on developing a conceptual understanding of both partitive and quotative division with a strong emphasis on visual fraction models. It is imperative that students understand how the operation of division works with fractions as opposed to memorizing a procedural rule. As students begin to further their understanding of computing with fractions to find quotients involving division of fractions by fractions, they should use visual models, including manipulatives and visual fraction models (e.g., number lines, area models, fraction bars) in order to interpret, represent, and solve mathematical and contextual problems. As students solidify their understanding of the division of fractions, they should be able to explain their reasoning in written and verbal form using models and diagrams to support their justifications.

It is imperative that students understand that division equations with fractions are interpreted the same ways as division equations with whole numbers from previous grades. For example, in the problem $\frac{1}{2} \div \frac{1}{6}$, students should understand that this division equation refers to either how many groups of $\frac{1}{6}$ are in $\frac{1}{2}$ or if $\frac{1}{2}$ is 1/6 of the group how much was the total group in the first place. To grasp this concept, students should begin with fraction models such as pattern blocks, visual drawings, and number lines because both situations are modeled very differently. By using manipulatives or visual representations in this problem, students can visually see that it takes 3 of the $\frac{1}{6}$ sized pieces to make up a $\frac{1}{2}$ sized piece of the same whole or that $\frac{1}{2}$ was $\frac{1}{6}$ of the group than the group was 3 to begin with. As students begin working with real-world problems, the situation in the problem will dictate which model is most appropriate.

Students should have equal practice with partitive division *(when you know the total number of groups and are trying to find the number of items in each group*) and quotative division (*when you know the number of items in each group and are trying to find the number of groups*) of fractions by fractions. As students

solidify their understanding, they should begin to move towards using the relationship between division and multiplication. Students should make the connection that $\frac{1}{2} \div \frac{1}{6} = 3$, and $3 \times \frac{1}{6} = \frac{1}{2}$. Students should use this understanding to make a connection that multiplication and division are inverse operations.

Level 4:

Students should extend their understanding of division of fractions by fractions by making the generalization that multiplication and division are inverse operations. As students have grasped a conceptual understanding of division of fractions by fractions they should be able to explain and interpret that $\frac{1}{2} \div \frac{1}{6} = 3$ because 3 is how many $\frac{1}{6}$ of $\frac{1}{2}$, which results in the multiplicative inverse of $\frac{1}{2} \times \frac{6}{1} = \frac{3}{1}$. This multiplication equation can be used to obtain the division equation directly, using the relationship between multiplication and division. As students deepen this understanding, they should be able to solve mathematical and contextual problems that involve partitive and quotative division problems, involving the division of fractions by fractions, and explain the solution using precise mathematical vocabulary.

The Number System (NS)

Standard 6.NS.B.2 Cluster Heading: B. Compute fluently with multi-digit numbers and find common factors and multiples.

Fluently divide multi-digit numbers using a standard algorithm.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use models to solve division problems with one and two-digit divisors. Know division is the inverse of multiplication. Know the place value of a number.	Connect multi-digit whole number division using a partial quotient algorithm to the standard algorithm for division with both one- and two-digit divisors. Determine the remainder in a multi-digit division problem and record with an R. Estimate the quotient in a multi-digit division problem.	Accurately, flexibly and efficiently divide multi-digit whole numbers using a standard algorithm. Apply place value understanding to divide multi-digit whole numbers. Interpret the quotient and remainder in a multi-digit division problem. Represent the remainder of a multi-digit division problem using a fraction or decimal.	Explain the division algorithm using place value language. Perform error analysis to critique a peer's division of multi-digit whole numbers.

Instructional Focus Statements

Level 3:

In grade 5 (5.NBT.B.6), students found whole-number quotients and remainders of whole numbers with up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. These strategies were highlighted with the use of area models to connect to multiplication and partial quotients as they relate to missing, decomposed factors in multiplication. Instruction in grade 6 should begin with highlighting these strategies and then explicit connections to the standard algorithm must be made.

In grade 6, students-are introduced to the standard algorithm and are expected to divide fluently with multi-digit whole numbers for the first time. It is important to understand fluency as not only operating accurately, but also flexibly, efficiently, and appropriately. In order to foster students' achievement of fluency, instructional strategies should focus on the meaning of division and its relationship to multiplication and place value. Additionally, divisors can be any number of digits at this grade level. As students divide, they should be expected to use their understanding of place value to describe what is happening using appropriate language. For example, when dividing 42 into 6,587, as they write a 1 in the quotient they should say, "there are 100 forty-twos in 6,587" rather than saying "there is one 42 in 65." It is important to ensure that students continue to apply their place value understanding to write 4200 beneath the 6,587 rather than only writing 42. Doing so will allow students to understand they are not simply "bringing down" the remaining digits when subtracting from the dividend. Discussion should also reveal that the initial 100 recorded in the quotient is just an estimate and that the quotient becomes more precise as they complete the division process. Previously students used visual and physical representations such as base ten blocks, area models, etc. Connections to these visual and physical representations of the division are crucial to develop a conceptual understanding of the procedure for division. In addition to finding the quotient, students should benefit from opportunities to discuss the meaning of the quotient and remainder. In grade 4 (4.NBT.B.6), students learned that the remainder could cause the quotient to increase by one, could be dropped or could be the answer to the question. This same understanding of the interpretations of the remainder are expected here. Students also previously recorded the remainder with an R, but in grade 6, should be expected to also write the remainder as a decimal or a fraction. Using the same example, further discussion will lead students to understand that there are 156 forty-twos in 6,587 but that there is still a remainder of 35. Realizing that there is not a full 42 left, students would write the remainder as 35/42 or approximately 0.83.

A solid understanding of the division algorithm with multi-digit whole numbers will prepare students to divide multi-digit decimals in standard 6.NS.B.3. Knowledge of this standard prepares students for A2.A.APR.A.1 where students are expected to divide polynomials using the long division algorithm.

Level 4:

At this level, students can not only fluently divide using the standard algorithm but can explain the process using place value language. They should be able to articulate their process and explain how the dividend is decomposed to break the division problem into smaller pieces by place value. When analyzing a peer's work, students at this level can perform error analysis to determine where a mistake was made in the division process and can articulate how to correct it. When discovering an incorrect quotient, they can use the relationship between multiplication and division to confirm the mistake but also use estimation to explain when a quotient is not reasonable. For example, consider a peer who says that 6,587 divided by 42 is 15.68. A student at this level could use estimation to show that this is not a reasonable quotient since 6000 divided by 40 is 150. Likewise, the student could also multiply 15.68 by 42 to show that the proposed quotient is much too small.

Students might also be asked to find the missing digits in a quotient, partial quotient, dividend or divisor at this level.

The Number System (NS)

Standard 6.NS.B.3 Cluster Heading: B. Compute fluently with multi-digit numbers and find common factors and multiples.

Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm and making connections to previous conceptual work with each operation.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Fluently add, subtract, and multiply whole numbers using strategies based on place value and properties of operations as well as the standard algorithm. Use models to add and subtract and multiply decimals to hundredths using strategies based on place value and properties of operations. Use models, place value strategies and properties of operations to divide whole numbers and decimals with a whole number divisor or dividend.	Estimate decimal numbers to add, subtract, multiply, and divide mentally. Divide multi-digit whole numbers using a standard algorithm. Use appropriate strategies and models to add and subtract, multiply, and divide decimals to hundredths with a whole number divisor or dividend.	Find a sum, difference, product, or quotient of multi-digit decimals flexibly, efficiently and accurately using a standard algorithm for each operation.	Using precise mathematical language, explain various parts of a model in relation to the standard algorithm. Use estimation to assess the reasonableness of the quotient in a division problem. Analyze student work and justify why an algorithm or strategy is correct or incorrect.

Instructional Focus Statements

Level 3:

Students extend their learning about the Number Base Ten System from prior grades, specifically standard 5.NBT.B.7, where they performed the four basic operations with whole numbers and decimals to the hundredths using models and strategies. Students are now expected to add, subtract, multiply, and divide with multi-digit decimals abstractly with the use of algorithms. It is important that this understanding be developed through conceptual understanding created from engaging in operations with explicit connections between the visual images and the abstract equations so that students make sense of the algorithms. By reasoning with place value, students apply their knowledge of the base ten number system to understand why numbers are placed to align the decimal points in addition and subtraction. For multiplication and division, students use reasoning of the base ten number system and knowledge of patterns involved when multiplying or dividing by powers of ten (5.NBT.A.2) to understand and explain the placement of the decimal in the product or quotient. Some students may not remember to use the concept of place value when adding tenths to hundredths. To help with this misconception, have students use manipulatives or drawings to show how the decimals have been added. Students should recognize that 0.5 is equivalent to 0.50. This focus on place value will help students with operations with decimals.

A student who has achieved fluency is able to work flexibly, accurately, and efficiently. To achieve fluency, students need sufficient, repeated, and on-going practice for each operation.

Level 4:

A student that demonstrates fluency should be expected to use estimation to approximate sums, differences, products, and quotients and use this estimation to assess the reasonableness of his or her answer. If a student's answer is not reasonable, then they should be able to locate and self-correct computational errors. Students should have opportunities to assess reasonableness of their own solutions as well as the solutions of their peers. Students should also be allowed to engage in error and strategy analysis to deepen their understanding of the operations.

The Number System (NS)

Standard 6.NS.B.4 Cluster Heading: B. Compute fluently with multi-digit numbers and find common factors and multiples.

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Informally explain the difference between factors and multiples. Find all factor pairs for a whole number in the range of 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a whole number in the range of 1-100 is a multiple of a given one-digit number. Determine whether a whole number in the range of 1-100 is prime or composite.	Evaluate expressions to determine if they are equivalent. List multiples of a given number from 1-12. List the factors for any whole number from 1 to 100. Use distributive property to write equivalent numerical expressions.	Rewrite the sum of two whole numbers using the GCF and distributive property. Determine the greatest common factor for two given whole numbers that are less than or equal to 100. Determine the least common multiple for two given whole numbers that are less than or equal to twelve.	Solve contextual problems that involve finding the greatest common factor. Solve contextual problems that involve finding the least common multiple.

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Instructional Focus Statements

Level 3:

In elementary grades, students worked with common factors and common multiples. In grade 4 (4.OA.B.4)., students wrote factor pairs to classify prime and composite numbers. In grade 5 (5.NF.A.1), students used common multiples to find common denominators when adding and subtracting fractions. In grade 6 that learning is extended as students find the greatest common factor (GCF) of two whole numbers less than or equal to 100 and the least common multiple (LCM) of two whole numbers less than or equal to 12. Students should focus on the relationship between the factors of numbers and use them to simplify and write equivalent numerical expressions.

Students are familiar with the distributive property, but this is their first experience with factoring to generate numerical expressions equivalent to the sum of two numbers. Students should have opportunities to practice finding the GCF and discuss how it is used it to write an equivalent expression using the distributive property. For example, when given then expression 27 + 45, students should identify that the greatest factor that both numbers have in common is 9. Therefore, the expression could be rewritten to highlight the GCF as 9 x 3 + 9 x 5 and can also be written as 9 (3 + 5). Students should be exposed to problems involving two prime numbers and conclude that the GCF of two prime numbers is 1.

Students are also expected to identify common multiples for two whole numbers. Students often state that 0 is not a multiple and 1 is a multiple of any whole number. To have a better understanding, students should have ample opportunities to discover and disprove these misconceptions. Students also confuse factors with multiples, so it is important that academic vocabulary is used consistently with regular practice. A thorough understanding of the factoring process using the GCF and distributive property will be essential as students go on to write equivalent algebraic expressions in standards 6.EE.A.3 and 7.EE.A.1.

Level 4:

To challenge students' understanding of LCM and GCF, they should be presented with contextual problems where they can apply their knowledge and skills to real-world problems. For example, "Hot dogs come in packages of 8, and hot dog buns come in packages of 12. How many packages of hot dogs, and how many packages of buns would you need to buy to have an equal number of hot dogs and buns?" Students should recognize that this problem indicates finding the least common multiple for both items and be expected to relate their answer back to the context of the problem.

The Number System (NS)

Standard 6.NS.C.5 Cluster Heading: C. Apply and extend previous understandings of numbers to the system of rational numbers.
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (*e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge*); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation as well as describing situations in which opposite quantities can combine to make 0.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define integers as positive or negative in a real-world context where the meaning of 0 is determined in the situation.	Identify opposite quantities used in real-world situations. Identify opposite quantities that combine to make 0.	Use integers to represent contextual situations involving quantities that have opposite directions or values. Explain the meaning of zero when used to represent quantities in real-world situations. Describe situations in which opposite quantities can combine to make 0.	Generate contextual situations involving rational numbers and their opposites. Create a real-world situation where opposites make zero.

Instructional Focus Statements

Level 3:

In elementary grades, students used positive rational numbers to represent contextual situations. Students are very familiar with the number line and the directionality of increasing and decreasing quantities. In grade 6, students will extend that understanding to continue the line beyond zero. In grade 6, they extend their learning to use integers to represent real-world situations and explain the meaning of 0 in these situations. For example, using −20 to represent a $20 debt or 20 to represent a $20 deposit into a bank account. For the first time, students are introduced to the concept of negative values.

Students should initially be exposed to negative numbers in contexts where it is natural to describe the magnitude (such as the vertical distance from sea level in meters) and direction quantity (above or below sea level) of the numbers. Students should have opportunities to engage in discourse around real-world contexts that involve positive and negative values and understand that some quantities can be measured in negative, or opposite values (i.e., temperature). Discussion should also include other quantities where this would not be appropriate, such as the number of football players on a team. Given a real-world context, students should explain the meaning of zero in each situation.

To describe situations in which opposite quantities combine to make zero, students might begin with two color counters where one color represents positive quantities, and one color represents negative quantities. An equal number of positive and negative counters represents zero. Ask students to make zero multiple ways. Students should also model on a number line how moving a certain amount in the positive direction following by moving the same amount in the opposite direction ends at zero on the number line.

Students are not expected to perform operations with negative numbers in grade 6, nor are they expected to define the complete Real Number System. However, a comprehensive understanding of numbers and their opposites will be beneficial as students learn to perform operations with negative numbers in grade 7 (7.NS.A.1) .

Level 4:

Students at this level have a strong understanding of how rational numbers are used to represent real-world situations. Students should be challenged to extend their learning to create contextual situations involving both positive and negative rational numbers. Students should continue discussions around the quantities used, and the meaning of zero in contextual situations using precise mathematical language.

The Number System (NS)

Standard 6.NS.C.6 Cluster Heading: C. Apply and extend previous understandings of numbers to the system of rational numbers.
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
6.NS.C.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, – (–3) = 3, and that 0 is its own opposite.*
6.NS.C.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
6.NS.C.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand that the number line can be extended to include negative numbers. Understand that the x-axis and y-axis can be extended to include negative numbers. Identify the graphed integers, given negative integers graphed on a number line. Locate positive rational numbers on a horizontal or vertical number line.	Locate positive and negative integers on a horizontal and vertical number line. Understand the connection between graphing on the number line to graphing on the coordinate plane. Graph ordered pairs of integers on a coordinate plane.	Describe positive and negative numbers as indicating opposite directions relative to 0 on the number line and explain the meaning of 0 in mathematical contexts. Locate positive and negative rational numbers on a horizontal and vertical number line. Recognize the opposite of the opposite of a number as the number itself.	Describe positive and negative numbers as indicating opposite directions relative to 0 and explain the meaning of 0 in contextual problems. Explain that changing the sign(s) of numbers in ordered pairs has the effect of reflecting the associated points across one or both axes.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Identify and locate points described by ordered pairs of positive and negative rational numbers on a coordinate plane.	

Instructional Focus Statements

Level 3:

In order to develop a strong conceptual understanding of negative numbers on the number line, students must first have a strong understanding of positive numbers and how they are used on both horizontal and vertical number lines. As students develop a conceptual understanding of integers, they should describe positive and negative numbers as indicating opposite directions relative to 0 on the number line and explain the meaning of 0 in contextual problems. Students should extend their understanding of locating integers on the number line to find and position rational numbers on vertical and horizontal number lines, including negative fractions and decimals. Students should make the connection that the opposite of a number on a number line is a reflection across 0. As students solidify their understanding of locating positive and negative integers on a number line, they should be able to transfer this knowledge to both vertical and horizontal number lines. Students should understand that the opposite of the opposite of a number is the number itself, including that 0 is its own opposite. This should be done with mathematical and contextual problems. It is important for students to describe quantities having opposite value to develop conceptual understanding of positive and negative numbers and their location on horizontal and vertical number line.

As students extend their understanding of locating points on a number line, students should transfer this knowledge with vertical and horizontal number lines to be able to plot points in all four quadrants of the coordinate plane. Mathematical discourse should lead students to make connections between graphing on vertical and horizontal number lines to graphing on the coordinate plane. Students extend their understanding of graphing points on the number line as reflecting across zero to graphing and reflecting points across the x- and y-axis, respectively. When students begin to plot points in all four quadrants they should develop an understanding that the signs of numbers in ordered pairs represents a singular location on the coordinate plane and that an ordered pair is composed of two parts. The first coordinate refers to the direction on the x-axis and the second part is the direction on the y-axis. Students should extend this knowledge to identify and locate ordered pairs containing positive and negative fractions and decimals. Locating and plotting points on the coordinate plane is imperative for future course work when students extend their knowledge of plotting points on a coordinate plane to graph equations.

Level 4:

As students deepen their understandings, they should be able to explain the connection that exists between horizontal and vertical number lines and the coordinate plane. Using a discovery learning approach, students should be able to make generalizations around the signs of ordered pairs within the four quadrants of the coordinate plane. Students should be able to identify which quadrant a point is located based on the signs of the order pairs. For example, an ordered pair where both signs are positive (+, +) lies in Quadrant I. This generalization should be coupled with the understanding that changing the sign of one or both numbers in the ordered pair will create a reflection of the point. As students make these connections and generalizations, students should display their knowledge with verbal and written explanation using precise mathematical language.

The Number System (NS)

Standard 6.NS.C.7 Cluster Heading: C. Apply and extend previous understandings of numbers to the system of rational numbers.
Understand ordering and absolute value of rational numbers.

6.NS.C.7a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

6.NS.C.7b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3° C > –7° C to express the fact that –3° C is warmer than –7° C.*

6.NS.C.7c. Understand the absolute value of a rational number as its distance from 0 on the number line and distinguish comparisons of absolute value from statements about order in a real-world context. *For example, an account balance of -24 dollars represents a greater debt than an account balance of -14 dollars because -24 is located to the left of -14 on the number line.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Compare two positive rational numbers without context using >, <, and = symbols to record the results of comparisons. Compare two rational numbers of opposite sign without context using >, <, and = symbols to record the results of comparisons.	Choose a representation showing a number line modeling absolute value as the distance a number is from 0. Compare two negative rational numbers using >, <, and = symbols to record the results of comparisons. Determine the absolute value of a rational number.	Compare two rational numbers using >, <, and = symbols to record the results of comparisons. Compare and order absolute values of rational numbers. Interpret statements of inequality as the comparison of two numbers on a number line diagram. Write, interpret, and explain statements of order for rational numbers in real-world contexts. Use a number line to model the	Interpret and order absolute values for rational numbers in real-world contexts. Recognize the relationship between absolute values and magnitude of quantities in context. Create contextual situations involving comparing and ordering rational numbers and explain the comparison using precise mathematical language. Create contextual situations that distinguish comparisons of absolute

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		absolute value of a rational number showing that it represents the distance the number is from zero. Distinguish comparisons of absolute value from statements about order in a real-world context.	value from statements about order.

Instructional Focus Statements

Level 3:

With the introduction of negative numbers, students gain a new sense of ordering on the number line. The focus of this standard is understanding the ordering and the absolute value of rational numbers and comparing them using <, >, and =. In previous grades, students ordered and compared positive numbers based on "more than" or "less than" a certain quantity. Transferring this knowledge to negative numbers requires closer attention to the position of the numbers on the number line. Students should develop a conceptual understanding of comparisons with negative numbers on a number line rather than their magnitude. When students compare positive numbers with inequality statements such as 7 > 3 on a number line, they see that 7 is further away from 0 and is greater than 3. When students compare negative numbers with inequality statements such as -7 < -3, it may be confusing that -7 is further away from 0 than -3. It is imperative for students to reason about comparisons with negative numbers referring to contextual situations to make sense that, in this case, -7 is less than -3. For example, as stated in the standard, students should understand that –7° *C* is colder than –3° *C* and the inequality comparison is -7 < -3. As this is the first-time students are introduced to absolute value, students should understand that absolute value is the distance from 0 on a number line. Students should model the absolute value of rational numbers on a number line to reinforce this understanding. The idea of absolute value should also be interpreted in real-world situations as magnitude to develop an in-depth conceptual understanding.

Level 4:

Students should extend their understanding by creating contextual situations that involve ordering rational numbers. This should be done by comparing rational numbers using inequality symbols and justifying their reasoning with verbal and written explanations, including using a number line. Students should also be able to explain the relationship between absolute values and magnitude of quantities in context. As students solidify their understanding of absolute value, they should be able to explain that as the value of a negative number decreases, its absolute value increases using precise mathematical language.

The Number System (NS)

Standard 6.NS.C.8 Cluster Heading: C. Apply and extend previous understandings of numbers to the system of rational numbers.
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Graph integers on a vertical and horizontal number line.	Use a number line to find the absolute value of a given number by determining its distance from zero. Given a set of coordinates, graph points on any quadrant of the coordinate plane.	Find the distance between two points on the coordinate plane using absolute value when both points lie on the same horizontal or vertical line. Solve real-world and mathematical problems by graphing points on the coordinate plane.	Generate contextual problems that involve graphing points on the coordinate plane.

Instructional Focus Statements

Level 3:

In grade 5 (5.OA.B.3b and 5.G.A.1), students plotted points in the first quadrant of the coordinate plane where coordinates were limited to positive whole numbers. In grade 6, students are introduced to negative numbers and therefore transition to locating and graphing points in all four quadrants on the coordinate plane. Students should have opportunities to graph points in the coordinate plane with the same x-coordinate or same y-coordinate and use them to draw horizontal and vertical line segments. Students are also expected to find the lengths of vertical and horizontal lines to solve real-world problems related to distance, segments and shapes. For example, using the vertices of a rectangle to determine the lengths of the line segments between them and using that information to solve for the perimeter or area of the rectangle.

Practice should include problems using coordinates that are in different quadrants in addition to being on the axes. Discussion should focus on how the process for finding the length of a line segment that lies on the x- or y-axis represents the distance between the two points, or absolute value. Students should recognize that this process remains the same when the line segment does not lie on an axes. When solving problems in the coordinate plane, students should realize that if they know the length of the line segment, then they can count up or down from the given end point to determine the coordinates of the other end point. Students may initially count the units between endpoints but should transition to interpreting distance between points in terms of absolute value. Experience graphing on the coordinate plane will be beneficial as students go on to analyze the relationships using graphs (standard 6.EE.C.9) and represent proportional relationships (standard 7.RP.A.2).

Level 4:

Students at this level have a strong understanding of absolute value and can explain how it relates to distance. They can not only solve problems by graphing, but also create real-world problems that involve graphing points on the coordinate plane using precise mathematical vocabulary. Students at this level should be challenged to include the origin and create problems that span across multiple quadrants. Students should also be challenged to create problems that could yield multiple solutions. *For example, the perimeter of a square is 36 units. One of the vertices of the square is located on the origin of a coordinate plane. One of the vertices is located in the 2nd quadrant. What are the possible coordinates of the vertices of the square?*

TN Department of Education

Expressions and Equations (EE)

Standard 6.EE.A.1 Cluster Heading: A. Apply and extend previous understandings of arithmetic to algebraic expressions.

Write and evaluate numerical expressions involving whole-number exponents.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Apply the order of operations to evaluate simple numerical expressions with grouping symbols. Recognize that the exponent denotes the number of times the base is used as a factor.	Apply the order of operations to evaluate numerical expressions with whole-number bases and whole-number exponents. Choose the numerical expression that models the context when given a real-world problem.	Write numerical expressions that represent a real-world or mathematical context with whole-number exponents and grouping symbols. Evaluate numerical expressions by applying the order of operations with whole-number exponents.	Write and evaluate complex numerical expressions that represent a real-world context with whole-number exponents including complex fractions. Create a real-world problem and the numerical expression that models the context using whole-number exponents including complex fractions.

Instructional Focus Statements

Level 3:

In grade 5, students were introduced to using the order of operations to evaluate whole-number numerical expressions (5.OA.A.1). Additionally, students had limited exposure to exponents as they only worked with situations involving powers of 10 (5.NBT.A.2). In grade 6, students write and evaluate more complex expressions including expressions which contain positive rational numbers (including positive fractions and decimals). Introductory instruction should focus on the meaning of the exponent as the number of times the base is used as a factor. Students should first experience practice with whole-number bases and then progress to positive fractions and positive decimals as the base. Manipulatives such as unit cubes can be used to show $3^2 = 9$

square units or model $3^3 = 27$ cubic units. A common misconception is for students to interpret 3 as 3 • 2 = 6. Writing the expanded notation of 3 = 3 • 3 as well as the use of a number line representation to model the expression can assist with addressing this common error.

Given verbal or visual representations, students should generate and evaluate numerical expressions that involve grouping symbols (parentheses, brackets, braces, and multiple sets of parentheses) as well as whole-number exponents. Students should also use the order of operations and/or properties of operations to determine a proper sequence of steps for evaluating expressions. Instruction should focus on applying order of operations as a process and not as a discrete set of memorized steps. This standard is not about teaching dependence on mnemonic phrases like PEMDAS but is about understanding the order of operations conceptually.

Students identify when the order of operations is incorrectly applied and understand that the placement of parentheses matters when evaluating expressions. Students should be presented with real-world problems that can be represented with numerical expressions. Additionally, students should engage in discussions about how each part of the expression relates to the given context. Students' ability to articulate the connections between the given context and expression will be critical when they later transition to algebraic expressions and equations. The initial use of simple contexts will allow students the opportunity to conceptually understand the connections between the context and the related numerical expression. The complexity of contexts should increase as students develop an understanding of the connections. Instruction for this standard should be integrated with standard 6.EE.A.2c.

Level 4:

Students should have opportunities to create real-world problems that represent a context based on numerical expressions. When writing expressions, students should be able to explain the connection that exists between the context and expression. When evaluating numerical expressions, students should have a deep understanding of order of operations and be able to determine when the order of operations has been applied correctly. Students should reason to determine when applying a property of operations would be more efficient. For example, if given 3.52 + 6 (20 + 2), applying the distributive property would be more efficient than finding the product of 6 and 22. Student explanations for their selected sequence of operations should be expressed in both verbal and written form using appropriate mathematical vocabulary.

Equations and Expressions (EE)

Standard 6.EE.A.2 Cluster Heading: A. Apply and extend previous understandings of arithmetic to algebraic expressions.
Write, read, and evaluate expressions in which variables stand for numbers.
6.EE.A.2a Write expressions that record operations with numbers and with variables. *For example, express the calculation "Subtract y from 5" as 5 - y.*
6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X*	X

*6.EE.A.2b only

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Apply the order of operations to evaluate algebraic expressions with whole numbers without exponents. Choose an expression using a letter to represent the unknown when given a verbal expression written in words such as "Subtract 2 from the product of 6 and y."	Apply the order of operations to evaluate algebraic expressions involving positive rational numbers without exponents. Read and identify the parts of an expression using appropriate mathematical vocabulary. Recognize that one or more parts of an expression can be written as single entity. For example, (8 + 7) can be written as a single quantity, 15.	Write expressions that record operations with numbers and with variables given a verbal expression written in words. Identify and describe parts of an expression using appropriate mathematical vocabulary. Evaluate algebraic expressions involving positive rational numbers with whole-number exponents using properties of operations and order of operations (when there are	Explain whether an algebraic expression has been correctly written to represent a given context and provide justification using precise mathematical language. Write a real-world problem to represent a given algebraic expression. Write algebraic expressions to generate formulas that arise from real-world problems (i.e., $P = 2l + 2w$).

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Write an expression using a letter to represent the unknown when given a verbal expression written in words such as “Subtract 2 from the product of 6 and y.”	no parentheses to specify a particular order). Substitute for the unknown (variable), when given a specific numerical value, to evaluate expressions including those that arise from formulas used in real-world problems.	

Instructional Focus Statements

Level 3:

Instruction for this standard should be integrated with standard 6.EE.A.1, where students used the order of operations and properties of operations to evaluate numerical expressions. This standard requires students to explore situations with an unknown represented by a variable. Students should be able to understand that a letter (variable) in an expression represents a number and when that letter is replaced with a number (substitution), the expression's value can be determined. When reading an expression, students should use the correct terminology for operations. For example, the expression $(x - 2) \div 4$ can be read as “the quotient of the quantity x reduced by 2 and 4." In addition to this understanding, students must also be able to write expressions from a description. For example, the quotient of the sum of x plus 4 and 2 is written as $(x + 4) \div 2$.

Students should be able to identify parts of an expression and accurately use mathematical terms such as constant, coefficient, variable, base, exponent, quantity, sum, difference, product, factor, quotient, and term when describing the expression. They should also understand that terms such as quantity, sum, difference, product, and quotient can indicate a single unit and frequently have implicit grouping symbols. Concrete manipulatives such as pattern blocks or algebra tiles can help students visualize the various parts of an expression.

Students should extend their understanding of substituting numerical values into algebraic expressions to solve real-world problems involving formulas such as $V = s^3$ and $P = 2l + 2w$. Students should also be able to substitute algebraic expressions to evaluate other unknowns. For example, when determining that the perimeter of a rectangular garden is the distance around the garden, students should discover that the sum of twice the length and twice the width will equal the perimeter of a rectangle, $P = 2l + 2w$. Given the width and length, students can substitute to evaluate the expression to find the perimeter. Additionally, students should realize that even though algebraic expressions have variables, the properties of operations still apply.

Level 4:

In addition to reading and evaluating expressions, students should be able to generate a real-world problem that can be represented by a given algebraic expression. Special attention should be given to the parts of the expression, ensuring that terms and operations accurately represent the context. When given an expression and a real-world problem, students should be able to determine if the expression accurately represents the context. If the expression does not accurately represent the context, they should be able to make revisions to the expression and explain why.

Equations and Expressions (EE)

Standard 6.EE.A.3 Cluster Heading: A. Apply and extend previous understandings of arithmetic to algebraic expressions.

Apply the properties of operations (including, but not limited to, commutative, associative, and distributive properties) to generate equivalent expressions. (The distributive property of multiplication over addition is prominent here. Negative coefficients are not an expectation at this grade level.) *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use the commutative or associative property to generate an equivalent expression. Name the properties of operations when given an example.	Apply the commutative, associative or distributive property to interpret and generate models to represent an equivalent expression.	Apply multiple properties of operations to rewrite an expression, generating an equivalent expression.	Perform error analysis and explain why two or more expressions are not equivalent using representations and appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

Beginning with grade 3 (3.MD.C.7), and further in grade 4 (4.NBT.B.5 and 4.NBT.B.6) and grade 5 (5.NBT.B.6 and 5.NBT.B.7), students used an area model to show the distributive property with whole numbers. This same model should be used to show the distribution of variables. For example, given a rectangle with a width is 4.5 units and a length of x + 3, the area of the rectangle can be expressed as 4.5 (x + 3) or 4.5x + 13.5. Students should understand that two or more expressions may be equivalent, even when their symbolic forms are different. When given an expression representing area, students need to find the factors. For example, the expression 10x + 15 can represent the area of a figure. Students find the greatest common factor (5) to represent the width and then apply the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would produce the equivalent expression of 5(2x + 3). Instruction should focus on making sense of the operation when interpreting expressions such as 3(x+5) as 3 groups of (x+5). Creating models is important to develop conceptual understanding. Students can model the distributive property with concrete manipulatives such as algebra tiles or with representations such as area models. A common mistake when applying the distributive property to an expression such as 3(x+5) is for students to forget to distribute the 3 to the second term within the parentheses. The use of manipulatives to develop a strong conceptual

understanding of the mathematics occurring within the distributive property helps students to recognize that 3(x+5) is equivalent to (x+5) + (x+5) + (x+5), which means that there are 3 x's and 3 5's resulting in 3x+15.

Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y must be 3y. They also use the distributive property of multiplication over addition, the multiplicative identity property of 1, and the commutative property for multiplication to justify that y + y + y and 3y are equivalent expressions.

For example, justify that y + y + y = 3y. Solution:

y + y + y
y • 1 + y • 1 + y • 1 using the Multiplicative Identity
y • (1 + 1 + 1) using the Distributive Property
y • 3 by sum of like terms
3y using the Commutative Property

As students solidify their understanding of the properties of operations, it is important that they can recognize and apply the properties to expressions. For example, students should recognize that (3 + 2) + 4 = 4 + (3 + 2) is an example of the commutative property. Likewise, students should recognize that both 2 x (4 + 1) = 2 x 4 + 2 x 1 and 3 x 7 + 3 x 5 = 3 x (7 + 5) are examples of the distributive property. A solid understanding of these properties is foundational for concepts such as factoring polynomials in high school courses.

Instruction for this standard should emphasize the application of properties of operations to generate and understand equivalent expressions, as opposed to the process of simplifying expressions. Students should demonstrate their understanding of equivalent expressions through various solution paths, defending answer choices, and critiquing reasoning of others. A firm grasp on variables as numbers helps students extend their work with the properties of operations from arithmetic to algebra. Properties have been introduced throughout grades 1-5 but emphasis has not been placed on naming the properties. Now students will use the name of the property to justify the computations needed to create equivalent expressions. Also note that negative coefficients are not an expectation in grade 6. Students will begin work with integer operations in grade 7.

Level 4:

At this level, students go beyond generating an equivalent expression and recognize that when the properties are applied correctly, they can generate multiple equivalent expressions. They can critique someone else's reasoning and explain why expressions are not equivalent. For example, given 2(x + 4) = 2x + 4, they can recognize that the distributive property has not been correctly applied and create a model to show why the two expressions are not equivalent. In addition to creating a model to show the error, they also use words to explain why the expressions are not equivalent. In this case, there would be 2 copies of x + 4, rather than 2 copies of x and 1 copy of 4. An equivalent expression would be 2x +8. Students can create the correct representation and recognize that the commutative property could be applied to write another equivalent expression of 8 + 2x.

Equations and Expressions (EE)

Standard 6.EE.A.4 Cluster Heading: A. Apply and extend previous understandings of arithmetic to algebraic expressions.
Identify when expressions are equivalent (i.e., when the expressions name the same number regardless of which value is substituted into them).
For example, the expression 5b + 3b *is equivalent to* (5 +3) b*, which is equivalent to* 8b.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an equivalent expression when the simplification requires combining like terms and does not require the application of a property of operations.	Choose multiple equivalent expressions when the simplification requires the application of a single property of operations. Generate equivalent expressions as a result of applying a single property of operations.	Choose multiple equivalent expressions when the simplification requires the application of multiple properties of operations. Identify when expressions are equivalent using substitution.	Generate equivalent expressions as a result of applying multiple properties of operations. Explain the process of generating equivalent expressions using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

Students should begin developing a conceptual understanding of combining like terms from previous course work with the distributive property (6.EE.A.3). For example, students should use properties of operations and combining like terms to justify that the expression 6a + 2a is equivalent to the expression a (6 + 2) as well as the expression 8a. Students should employ this previous understanding coupled with utilizing properties of operations to rewrite expressions. In doing this, students should discover that expressions written in different forms produce equivalent expressions. Students should understand that two or more expressions may be equivalent, even when their symbolic forms different. Specifically, students should use the associative, commutative, and distributive properties to produce equivalent expressions.

Students should also prove equivalence of expressions using substitution. For example, prove that the expressions 3(2 + x) and 6 + x are equivalent by substituting the same value for x in each expression. If expressions result in the sale quantity, they are equivalent. If they result in different quantities, they

are not equivalent. Students should conceptually understand that two or more expressions are equivalent if they have the same value, regardless of what number is consistently substituted for all values of a particular variable.
Instruction for this standard should emphasize the use of properties of operations and substitution to identity equivalent expressions, as opposed to the process of simplifying expressions. Concrete manipulatives such as colored squares or algebra tiles can help students visualize various forms of equivalent expressions.

Level 4:

Students should move from identifying equivalent expressions to generating equivalent expressions as a result of applying multiple properties of operations and explain the connection between the expressions. This standard lays the foundation for future course work where students rewrite expressions to reveal specific quantities. Additionally, students should solidify this understanding by explaining their reasoning with precise mathematical vocabulary.

Equations and Expressions (EE)

Standard 6.EE.B.5 Cluster Heading: B. Reason about and solve one-variable equations and inequalities.

Understand that a solution to an equation or inequality is the value(s) that makes that statement true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain what equality and inequality symbols represent. Determine if a numerical equation or inequality is true or false.	Use substitution to determine if a value from a given set is the solution to an equation that contains variables.	Use substitution to determine if a value from a given set is the solution to an equation or inequality that contains variables.	Generate a model to show that a value makes an equation or inequality true or false. Reason to predict whether a given value is the solution to an equation.

Instructional Focus Statements

Level 3:

Standard 6.EE.A.2 requires students to evaluate expressions in which variables stood for numbers, so students understand that a number can be substituted for a variable to find the value of an expression. This experience with expressions prepares students for work with equations and inequalities. For mastery of this standard (6.EE.B.5), students should understand that an equation is a mathematical statement formed by setting two expressions equal to each other. They should also understand that an inequality is a set of two expressions, where one expression is greater than (or greater than or equal to) the other.

Equations are true and valid mathematical statements if the two expressions have the same value. Students should understand that finding the solution to an equation requires them to find the value that can be substituted for the variable to make the statement true. If when substituting a number does not make the statement true, they understand that the number is not a solution. For example, given 7 + s = 23 and a solution set of 4, 3 ½, 16, they understand that substituting a 4 for s would give one expression a value of 11 and the other expression a value of 23. Since substituting a 4 does not make the

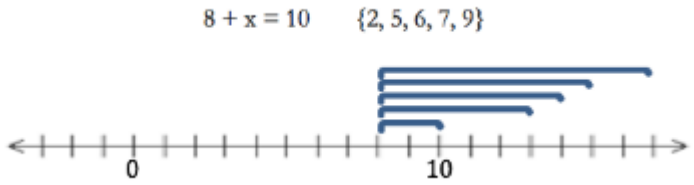
statement true, it is not a solution. Similarly, when 16 is substituted for s, both expressions have a value of 23, meaning the two expressions are equivalent and the statement is true. Additionally, they can write the solution to the equation as a simpler equation, s = 16.

As students explore solutions to inequalities, they should be given solution sets where multiple values satisfy the inequality. From these experiences, they should conclude that inequalities can have multiple solutions. For example, given 4.5x > 22.5 and a solution set of 0, 6, 10 , 16 , students should understand that 6, 10, and 16 are all solutions since they all make the inequality true when substituted for x.

At this point, students only work with one-step equations and inequalities, and are not required to use a formal method for solving them. It is not yet necessary for them to use inverse operations to solve equations or inequalities. The focus for this standard should be answering the question, "Which values from a specific set make the equation or inequality true?" Coefficients, constants and solutions are positive rational numbers, including fractions and decimals. Both equations and inequalities can contain variables and constants. A solid understanding of equations, inequalities and their solutions will prepare students to formally solve equations in standard 6.EE.B.7.

Level 4:

In addition to using substitution to determine if a given value is a solution to an equation or inequality, students should be able to create a model, such as a number line, to show that the value is a solution (see model below). In this example, a number line is used to prove that 2 is the only possible solution to the equation, 8 + x = 10. All other values beyond 2 yield a value not equal to 10.



Given an equation or inequality, students should reason about the values within the solution set. For example, given ¼ x = 8 and a solution set of 0, 2, and 4, students should reason that the solution cannot be 0 because ¼ of 8 has some value greater than 0. They should also reason that the solution cannot be 4 because 4 is ½ of 8. Substitution should be used to verify the solution.

At this level, students reason about the solution to an equation without needing to substitute. Given 8 = ½ (x), student's reason that there can only be one value that satisfies this equation. They also realize that the solution must be 16 since that is the only number that one can take half of to get 8.

Equations and Expressions (EE)

Standard 6.EE.B.6 Cluster Heading: B. Reason about and solve one-variable equations and inequalities.

Use variables to represent numbers and write expressions when solving a real-world and mathematical problems; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify variables in an expression. State the meaning of a variable.	Use variables to write expressions and solve mathematical problems.	Use variables to write expressions and solve real-world problems. Explain the meaning of a variable in an expression using precise mathematical vocabulary.	Generate a contextual problem to represent the relationship in an algebraic expression.

Instructional Focus Statements

Level 3:

This standard is a continuation of grade 6 coursework related to expressions. In standard 6.EE.A.2, students identified the different parts of an expression and evaluated them given a value for the variable. They also translated numerical and algebraic expressions from verbal representations. Students should now move beyond mathematical translations to interpreting context to write algebraic expressions involving multiple operations. However, students in grade 6 should be limited to problems involving nonnegative rational numbers.

Students are progressing in their ability to represent situations with expressions and show the relationship between quantities in the problem. Students should be presented with real-world problems and be provided opportunities to engage in discussion about how the context relates to each part of the written expression. The initial use of simple contexts will allow students to conceptually develop an understanding of the connections that exist between the context and the algebraic expressions. Additionally, the use of models and diagrams will aid students in making sense of the operations used to solve the problem.

Students should precisely explain the meaning of the variable used in the expression and determine when the variable represents:

- A single number, often when the expression can be written as an equation.
  For example: Mr. Smith is buying cupcakes in cartons that hold a dozen cupcakes. If he is buying cupcakes for 48 students, how many cartons will he need? We could say the number of cartons needed could be represented with the expression 48/12 or the equation, c = 48/12.
- All numbers, such as in an expression.
  For example: Tina's puppy weighs 14 ounces more this week than last week. Write an expression to represent puppy's weight this week. p + 16 when p is the puppy's weight this week.
- A range of numbers.
  For example: Mr. Smith is buying cupcakes for his students in cartons that hold a dozen cupcakes. Write an expression that represents the largest number of students that can receive a cupcake in *c* cartons. This situation would be represented with the expression 12c.

A solid understanding of writing expressions will prepare students to write and solve equations (standard 6.EE.B.7).

Level 4:

Students should be able to generate a real-world problem from an algebraic expression and explain the parts of the expressions in terms of the context. Given an expression such as 5n + 100, students should write context that represents the expression given. For example, it costs $100 to rent the skating rink plus $5 per person. Write an expression to find the cost for any number, *n*, of people. Students should be challenged to not only generate context, but also explain the meaning of the solution in terms of the context using precise mathematical vocabulary. In this situation, discussion should include the meaning of the constant, the variable, the coefficient, the addition operation and how each of these relates to the context created by the student.

Equations and Expressions (EE)

Standard 6.EE.B.7 Cluster Heading: B. Reason about and solve one-variable equations and inequalities.

Solve real-world and mathematical problems by writing and solving one-step equations of the form $x + p = q$, $px = q$, $x - p = q$, *and* $x/p = q$ for cases in which p, q, and x are all nonnegative rational numbers and $p \neq 0$. (Complex fractions are not an expectation at this grade level.)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve an equation in the form $x + p = q$ or $x - p = q$ when p, q, and x are all whole numbers.	Identify the variable quantity in a real-world or mathematical situation. Choose an appropriate equation to model a given situation or real-world problem. Solve equations of the form $px = q$ *or* $x/p = q$ when p, q, and x are all whole numbers.	Solve equations of the form $x + p = q$, $px = q$, $x - p = q$, *or* $x/p = q$ when p, q, and x are all non-negative rational numbers. Write equations in the form $x + p = q$, $px = q$, $x - p = q$, *or* $x/p = q$ to represent real-world contexts.	Explain the relationship between the context and the written equation in both verbal and written form.

Instructional Focus Statements

Level 3:

In grades K-5, students experienced writing numerical equations and using symbols to represent unknown numbers. An equation is a statement that shows two equivalent expressions using an equal sign. This standard builds on 6.EE.B.5 where students learned that a solution to an algebraic equation is the value(s) that makes the statement true. Solving equations should be approached as a process of reasoning rather than a set of steps to follow. This understanding can be reinforced by comparing arithmetic and algebraic solutions to real-world problems. For example, how many $8 movie tickets can

you buy with $48? Students could solve this problem using simple division but should now begin to see how to represent the problem algebraically as $8t = 48$.

To solve an equation, students perform operations to the equation so that one expression is left with only the variable and the other expression is left with only a numerical value. For example, solving the equation $2x = 6$ means that students are being asked to find the value for the variable x for which the expression $2x$ has the same value as the expression 6. Students might multiply both expressions by ½, resulting in $x = 3$. Students could also divide both expressions by 2, also resulting in $x = 3$. Using substitution (6.EE.A.4), students can confirm that $2 \bullet 3 = 6$.

Students should develop a conceptual understanding of solving one-step equations involving positive rational numbers (including zero), fractions, and decimals. Instruction should be focused on real-world problems with students generating equations based on the given situations. Students should illustrate the equation in problem situations with visual representations including drawing pictures, diagrams, bar models, etc., and use reasoning and prior knowledge to solidify their understanding.

Level 4:

As students deepen their understanding of solving one-step equations resulting from real-world situations, they should not only create an equation from a real-world or mathematical situation but also identify and interpret dependent and independent variables with respect to the context. Solving equations is a process of reasoning to find the number(s) which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should be flexible working with different examples where looking for structure will produce more efficient solution paths. This allows them to explain their reasoning for selecting the specific solution path.

Equations and Expressions (EE)

Standard 6.EE.B.8 Cluster Heading: B. Reason about and solve one-variable equations and inequalities.

Interpret and write an inequality of the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$ which represents a condition or constraint in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions; represent solutions of inequalities on number line diagrams.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a value that makes a given inequality true.	Choose a number line graph that represents a given inequality in the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$. Choose an inequality in the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$ that represents a constraint or condition in a given real-world or mathematical problem or situation.	Write an inequality of the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$ to represent a constraint or condition in a real-world or mathematical situation. Graph an inequality in the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$ on a number line. Write an inequality in the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$, given an inequality graphed on a number line.	Generate multiple values that make an inequality true. Write an inequality $x > c$, $x < c$, $x \leq c$, or $x \geq c$ to represent a constraint or condition in complex real-world and mathematical problems.

Instructional Focus Statements

Level 3:

Students have previous experience with the vocabulary of greater than and less than as well as use of the symbols > and < in grades K-5. Students should build on their previous understanding to write inequalities in the form of $x > c$, $x < c$, $x \leq c$, or $x \geq c$. The use of ≤ and ≥ are new to students in 6th grade. Students should be able to interpret and write inequalities in the given forms to represent a constraint or condition in a real-world or mathematical

situation. Through reasoning, students determine the appropriate inequality to use in a given situation. For example, a person must be at least 16 years old to obtain a driver's license might be represented by the inequality $a \geq 16$.

The focus of this standard is also for students to graph inequalities on a number line. Students should understand the difference between $>$, $\geq$, and $<$, $\leq$ and graphing with the appropriate open or closed circle. Students should discover that a variable can stand for an infinite number of solutions when used in inequalities and the need to shade the number line to represent these infinite solutions. Students should also be able to write an inequality from its graphical representation on a number line. Students should use this knowledge to then check solutions using substitution to determine if the values satisfy an inequality, building on understanding from standard 6.EE.B.5.

In 6th grade, students are not expected to solve inequalities or to write compound inequalities. Students will build on this work in 7th grade (7.EE.B.4b) when they solve contextual problems leading to inequalities.

Level 4:

As students solidify their foundational understanding of interpreting and writing inequalities, they should extend their knowledge to generating inequalities from a context and creating a context from a given inequality. Students should also combine these skills to graph their solutions on a number line and understand what the solution set represents with respect to the context.

Equations and Expressions (EE)

Standard 6.EE.C.9 Cluster Heading: C. Represent and analyze quantitative relationships between dependent and independent variables.
Use variables to represent two quantities in a real-world problem that change in relationship to one another.
For example, Susan is putting money in her savings account by depositing a set amount each week ($50). Represent her savings account balance with respect to the number of weekly deposits (s = 50w, illustrating the relationship between balance amount s and number of weeks w).
6.EE.C.9a Write an equation in the form y = px where y, p and x are all non-negative and p ≠ 0, to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.
6.EE.B.9b Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the value of the dependent variable given the value of the independent variable given an equation in the form y = px.	Identify the dependent and/or independent variables in a real-world situation. Choose a graph or table that represents a given two-variable equation in the form y = px.	Determine independent and dependent variables and write an equation in the form y = px that represents the relationship. Recognize that a change in the independent variable creates a change in the dependent variable. Analyze relationships between variables using tables, graphs, and equations.	Analyze the relationship between dependent and independent variables in complex real-world situations using equations, graphs, and tables.

Instructional Focus Statements

Level 3:

Students should begin developing a conceptual understanding of using variables to represent quantiles in simple real-world situations. This will be a building block for future work with expressions, equations and functions. Building on writing and solving equations in one variable (6.EE.B.7), students will use two variables to express relationships between two quantities that vary together. It is imperative that students recognize that a change in the independent variable creates a change in the dependent variable, such that as x changes, y also changes. A focus is placed on writing an equation to represent the quantities in terms of the dependent variable and independent variable. Additionally, students should employ this understanding to analyze the relationship between the variables using graphs and tables and make connections back to the equation. Ensure that students understand that each representation shows the same relationship. These equations, tables and graphs should represent relationships with positive slopes only.

Level 4:

Students should move from using variables to represent two quantities to analyzing the relationship between the quantities in complex real-world problems with graphs, tables, and equations. Students should also provide written and verbal justification to explain the relationship of the quantities and the connections between multiple representations. It is also important for students to understand and explain that each representation (graph, table, equation) shows the same relationship. Students will extend their understanding of the relationship between independent and dependent variables in later grades as they work with linear functions that are both discrete and continuous.

Geometry (G)

Standard 6.G.A.1 Cluster Heading: A. Solve real-world and mathematical problems involving area, surface area, and volume.

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; know and apply these techniques in the context of solving real-world and mathematical problems.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the area of right triangles by composing into rectangles, then find the area of the rectangles. Find the area of right triangles and rectangles in contextual problems.	Find the area of a right triangle. Find the area of a quadrilateral given its decomposition into rectangles and/or right triangles. Find the area of non-right triangles given their decomposition into right triangles.	Find the area of non-right triangles by decomposing them into right triangles. Find the area of quadrilaterals and polygons that can be composed into rectangles and/or decomposed into triangles and other shapes. Find the area of polygons embedded in real-world and mathematical problems by composing and/or decomposing them into simpler shapes.	Find the area of complex polygons by composing or decomposing them into simpler shapes. Find the area of polygons embedded in complex real-world and mathematical problems.

Instructional Focus Statements

Level 3:

Students should work with drawings and models/manipulatives to develop a conceptual understanding that familiar shapes can be put together to create composite shapes, and unfamiliar shapes can be decomposed into non-overlapping, adjacent familiar shapes. In grade 3, students developed an understanding of how to find the area of a rectangle using an area model and generalized this concept to discover the area formula for a rectangle. In grade 6, students should employ their understanding of finding the area of a rectangle to then decompose a rectangle into two right triangles to discover

that the area of a right triangle is half the area of a rectangle. Therefore, discovering that the area of a right triangle is $A = \frac{1}{2} bh$. Note that students commonly learn the formula for area of a rectangle as *length x width*, while a triangle is *one-half x base x height*. Therefore, to avoid confusion with the vocabulary, it is important for students to understand that in reference to a rectangle the terms length and width can also be referred to as base and height.

Students should use manipulatives or paper shapes to physically compose and decompose triangles, quadrilaterals, and polygons to help them develop a concrete conceptual understanding of the process before being asked to solve using drawings or representations. For example, before giving students the formula to find the area of a triangle, hand students a paper rectangle and scissors and ask them to cut along a single line to turn the rectangle into triangles. Students can label the edges of a paper rectangle and use scissors to physically cut along a diagonal to see the decomposition into two right triangles with the base and height (and/or length and width) labels still visible. Therefore, they can see that the base and height are still intact, but each right triangle is exactly half the area of the rectangle.

This same activity can be done with a parallelogram that is not a rectangle to see that cutting along the diagonal results in two non-right triangles that can then each be decomposed into two right triangles. Students should be allowed to explore different ways to compose and decompose quadrilaterals and other polygons. For example, parallelograms can also be composed into a rectangle by cutting off a right triangle from one end, rotating it 180 degrees and reattaching it to the opposite end. This means that the area of the parallelogram can also be found using same formula as the rectangle by multiplying the length of the base by the height of the right triangle, which is also the height of the parallelogram and the resulting rectangle.

Allowing students the opportunity to explore area using these manipulatives will help students avoid the confusion that commonly occurs when students are distinguishing between area and perimeter. Students should expand on this knowledge, coupled with the understanding of composing and decomposing shapes, to build an understanding that the area of composite shapes is equal to the sum of the areas of the non-overlapping, adjacent shapes.

Level 4:

Students should interchangeably compose and decompose shapes to form familiar shapes. They use these to find the area of familiar shapes and the total area of the original composite shape. Students at this level should be challenged with finding the area of polygons of increasing complexity. It is important that students are able to verbalize that these are equivalent and explain why. In solidifying this concept, students should generalize that the area of a right triangle is half the area of a rectangle resulting in the formula $A = \frac{1}{2} bh$ and explain this connection using precise mathematical vocabulary. Additionally, as students work with the same shape, they should be able to compare different decompositions to see that a shape can be decomposed in multiple ways and have the same area. This understanding should be transferred into solving mathematical and real-world problems by using precise mathematical vocabulary to explain their reasoning.

TN Department of Education

Geometry (G)

Standard 6.G.A.2 Cluster Heading: A. Solve real-world and mathematical problems involving area, surface area, and volume.

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ where B is the area of the base to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the volume of a right rectangular prism with whole-number side lengths using a visual model. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and equate this to finding volume by multiplying the edge lengths.	Find the volume of a right rectangular prism represented visually with a single simple fractional edge length. Model the volume of a right rectangular prism when a single side has a fractional edge length using unit cubes with dimensions of the appropriate unit fraction.	Find the volume of right rectangular prisms with fractional edge lengths to solve real-world and mathematical problems when a visual model is provided. Model the volume of a right rectangular prism with multiple fractional edge lengths using unit cubes with dimensions of an appropriate unit fraction. Show that the volume found by creating a model of a right rectangular prism packed with unit cubes is the same as would be found by multiplying the edge lengths of the prism. Explain the relationship between the formulas for calculating volume	Solve real-world and mathematical problems by finding the volume of right rectangular prisms with multiple fractional edge lengths when a visual model is not provided. Explain why the volume found by creating a model of a right rectangular prism is the same as would be found by multiplying the edge lengths of the prism.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		$V = lwh$ and $V = Bh$.	

Instructional Focus Statements

Level 3:

In grade 5, students developed an understanding of volume as the filing of a space with unit cubes with whole number edge lengths and generalized the volume formula for a right rectangular prism in standard 5.MD.C.5. They also further developed their understanding of operations with fractions in the NF domain. Grade 5 students were expected to "know", i.e. commit to memory, the formula for finding the volume of a right rectangular prism. In grade 6, students will combine these concepts and extend their understanding to find the volume of a right rectangular prism with fractional edge lengths.

Students should connect back to their work in grade 5 and model volume by filling or building a rectangular prism with unit cubes, but now using unit cubes with fractional edge lengths based on the same unit fraction and use the model to determine the volume. Students should be able to show that filling or building a solid with unit cubes and counting them gives the same result as using the volume formula.

Students are still developing their fractional computation skills and may have deficits. Instruction should be differentiated to provide students with opportunities to improve their fractional computation fluency.

It is important for students to understand that an upper-case B represents something different than a lower-case b in geometry formulas. Specifically, B represents the area of the base, while b represents an edge length. Instruction should emphasize the connection between the two volume formulas to help students see that since the shape of the base of a rectangular prism is a rectangle and one way to find the area of that base is length x width ($l \times w$), then $V = lwh$ can be rewritten as $V = Bh$, because lw is equivalent to B for this solid.

The formula $V = Bh$ is foundational for students as they will use the same formula to find the volume of prisms with differently shaped bases in grade 7. Therefore, understanding $V = Bh$ as "the area of the base times the height of the prism" as a formula for finding the volume of a rectangular prism is vital for students to later broaden their understanding that finding B will depend upon the shape of the base of the prism.

Level 4:

Students at this level demonstrate a strong conceptual understanding of volume using fractional computation skills and modeling skills. Students should employ this understanding to solve complex real-world problems. Students should also be able to model their thinking with multiple representations. Additionally, students should provide written and verbal justification explaining the connections between multiple methods of finding volume.

Geometry (G)

Standard 6.G.A.3 Cluster Heading: A. Solve real-world and mathematical problems involving area, surface area, and volume.

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side that joins two vertices (vertical or horizontal segments only). Apply these techniques in the context of solving real-world and mathematical problems.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Accurately plot points in a coordinate plane. Find the distance between two points on a number line.	Find the length of a vertical or horizontal side of a polygon when provided a visual representation of the polygon on a coordinate plane.	Draw a polygon in the coordinate plane given a set of coordinates for the vertices. Find the lengths of the sides of a polygon with horizontal and vertical sides. Use a polygon drawn in a coordinate plane with horizontal and vertical sides to solve real-world and mathematical problems.	Create a real-world problem that involves drawing a polygon in the coordinate plane and use precise mathematical language to explain the solution. Find the lengths of the sides of a polygon given the coordinates of the vertices without graphing and explain their reasoning.

Instructional Focus Statements

Level 3:

In grade 5, students begin exploring and plotting points in the first quadrant on a coordinate plane (5.G.A.1). This is extended to all four quadrants in grade 6 (6.NS.C.6). In this standard, students begin to explore polygons that are graphed in a coordinate plane with sides that are either vertical or horizontal. This also connects to the concept of using absolute value to find the distance between two points with either the same x or y coordinate (6.NS.C.8). Calculating lengths of line segments that are not vertical or horizontal will be addressed in grade 8. Instruction should guide students to connect finding the lengths of horizontal and vertical line segments in a coordinate plane to finding the distance between two points on a number line. To develop a

conceptual understanding, students should use graph paper to determine the lengths of the sides by counting the number of units from one vertex to another. This strategy should be applied to real-world and mathematical problems.

A common misconception occurs when students are unclear about what they are counting. Students with this misconception may count the markings, rather than the spaces between the markings, which leads to an incorrect length. Connecting this back to the distance between points on a number line and guiding students to recognize the relationship between finding distance and subtraction can help students better understand the concept of a length.

It is important for students to understand the usefulness of finding lengths in a coordinate plane through real-world application. For example, students may be asked to find the perimeter of a garden to find how much fencing is needed to enclose it by graphing a diagram on a coordinate plane. This can be especially helpful if the garden has more than four sides because it stretches around two sides of a building. Students should be given the opportunity to explore different polygons and not be limited to quadrilaterals. However, when considering a square or a rectangle, students should use what they know about the attributes of these figures to explain how they know each figure is a square or a rectangle.

This standard prepares students for work on scale drawings and constructions in the 7^{th} grade.

Level 4:

As students strengthen their understanding, they should make connections about how to find the length of the sides of a polygon using the coordinates of the vertices having the same first coordinate (or second coordinate) and generalize their thought process. This should be accompanied by mathematical justifications extending from a polygon drawn in the coordinate plane and not a stand-alone algorithm. It is important for students to recognize and explain the connection between finding the side length of a polygon on a coordinate plane to distance on a number line and absolute value to solidify their conceptual understanding. As students develop a strong understanding of finding the side lengths of a polygon, they should be able to tackle more rigorous problems. For example, when given all but one coordinate for the vertices of a regular polygon, the student can identify the missing coordinate and then graph the polygon on the coordinate plane. Students should reinforce their understanding of drawing polygons and finding the length of the sides of a polygon in the coordinate plane by solving contextual problems and providing explanations in written and verbal form using precise mathematical vocabulary.

Geometry (G)

Standard 6.G.A.4 Cluster Heading: A. Solve real-world and mathematical problems involving area, surface area, and volume.

Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the net of a rectangular prism given a visual model. Calculate the area of a rectangle. Calculate the area of a triangle.	Identify the net of a triangular prism given a visual model. Identify the net of a rectangular pyramid or a triangular pyramid given a visual model.	Draw a net to represent prisms and pyramids made up of rectangles and triangles. Use nets of three-dimensional figures to calculate the surface area. Relate the area of two-dimensional figures to find the surface area of three-dimensional figures. Solve real-world and mathematical problems involving surface area of prisms and pyramids made up of rectangles and triangles.	Explain similarities and differences between calculating the surface area and the volume of a solid using precise math vocabulary. Create real-world scenarios that involve finding surface area of three-dimensional figures. Explore nets of other solid figures made up of rectangles and triangles to find their surface areas.

Instructional Focus Statements

Level 3:

In grade 6, students expand on their prior understanding of the area of two-dimensional figures to finding the surface area of three-dimensional figures. Students should begin concretely exploring the concept of surface area by unfolding prisms and pyramids into nets and folding nets into prisms and pyramids. Instruction should allow students to make connections between the appearance of the net and the prism, paying particular attention to the

shape of each face and where each face falls, starting with rectangular prisms and moving to triangular prisms, rectangular pyramids, and triangular pyramids.

Students should work with physical models and eventually visual representations to gain a conceptual understanding that surface area is composed of the sum of the areas of the faces. This standard is not about memorizing formulas. It is about students developing a strong conceptual understanding of what surface area is. Therefore, the focus for calculating it should be by adding together the set of rectangular and/or triangular areas that make up the faces of the solid figure. The formulas will be addressed in a future course.

Students should be exposed to a wide variety of three-dimensional shapes with sides composed of rectangles and triangles. Allowing students the opportunity to explore the concept of surface area both by unfolding physical models of prisms and pyramids into their nets as well as folding given nets into the solids will help students not only make connections between the nets and their solids but will also help students begin to recognize properties of each figure. In strengthening this understanding, students should explain their processes using precise mathematical vocabulary such as face, vertex, edge, base, area, surface area, prism, and pyramid. As students unfold a solid into its net, they should draw the net as a set of rectangles and triangles. Students should be encouraged to consider the different ways a net can be drawn for the same figure. By the end of the course, students should be able to identify and draw nets associated with three-dimensional figures and use this to find the surface area of the figure in mathematical and real-world mathematical problems.

As students deepen their understanding of surface area, they should be able to explain the connections between how the area of a two-dimensional figure relates to the surface area of a three-dimensional figure. As students make these connections, they look for and make sense of structure (MP7).

Level 4:

Once students have a clear understanding of identifying nets of prisms and pyramids, students can begin to explore the nets of other three-dimensional figures with faces that are rectangular, triangular, or that can be broken down into rectangles and triangles. Students can then be challenged to find their surface areas. Students can also be challenged to create their own real-world scenarios that involve finding surface area.

Students at this level should be able to describe similarities and differences between finding surface area and volume of a solid, comparing the definitions and processes for calculations using precise mathematical vocabulary.

Statistics and Probability (SP)

Standard 6.SP.A.1 Cluster Heading: A. Develop understanding of statistical variability.

Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define data. Collect data from various sources.	Recognize variability in data. Define statistical question.	Distinguish between a statistical and a non-statistical question. Write statistical questions. Collect data to answer statistical questions.	Write statistical questions that can result in a wide range of responses.

Instructional Focus Statements

Level 3:

In grades 4 and 5, students learned to represent data using line plots. In grade 6, they move beyond representing given data to generating statistical questions and analyzing data. Students should be presented with multiple examples of statistical and non-statistical questions and encouraged to think of possible answers to the questions. Opportunities for discourse should reveal that the answers to statistical questions vary from one individual to the next. For example, "How many books did you read this month?" is not a statistical question because there is only one answer. However, "How many books did each student in your class read this month?" is a statistical question as it anticipates variability in books read by each student. Students should realize that statistical questions require a collection of data. To help students distinguish between the question types, encourage them to think of possible data values and consider how they vary. Students should also engage in activities that require them to generate statistical questions and collect data to answer the questions. Facilitated discussion should lead students to understand the concept of variability in the data and know that not all data values will be the same for a statistical question. An understanding of statistical questioning and data variability will be essential as students go on to use data to draw inferences in grade 7.

Level 4:

Students at this level can explain how to differentiate between statistical and non-statistical questions using precise vocabulary that describes the difference in data variability. Students also understand that statistical questions can have a narrow or wide range of values. They can generate statistical questions that can result in a wide range of values. For example, rather than asking classmates "How many days have students in my class been absent this week?" they might ask, "How many days have students in my class been absent this month?" or even "How many days have students in my class been absent this quarter?" They realize that absences over a month or quarter will have a greater variation between observations.

Statistics and Probability (SP)

Standard 6.SP.A.2 Cluster Heading: A. Develop understanding of statistical variability.

Understand that a set of data collected to answer a statistical question has a distribution which can be described by its measures of center (mean, median, mode), measures of variation (range only), and overall shape.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Collect data to answer a statistical question. Display single, numerical data sets with line plots.	Calculate the mean, median, mode and range of a given set of data. Display single, numerical data sets with box plots, stem plots, and histograms.	Use the mean, median, and mode to describe data that has been generated from a statistical question. Use the range to describe data that has been generated from a statistical question. Describe the overall shape of a data set with respect to its center.	Determine whether median or mean is a better choice for describing the center of a set of data collected from a statistical question. Describe the variation of a data set using the range. Describe the impact of an extreme value on the median, mean, and range.

Instructional Focus Statements

Level 3:

In grade 5, students created line plots to display numerical data sets and solved problems involving information presented in line plots. In grade 6, students build on that learning to display numerical data using several types of graphs, including box plots and stem plots (6.SP.B.4). This standard requires students to go beyond displaying data to describing data generated from a statistical question. They will describe the data using the measures of center (mean, median, mode), the measure of variability (range) and the overall shape. Students should be expected to describe and analyze raw data and data distributions using graphical displays of data. When presented with a graphical display of data, students should engage in discourse around what

information can or cannot be determined from the data. Students should summarize the data with a single value for each of the measures of center (mean, median, and mode) as well as the measures of variability (range). When describing the center, variability and shape of data, students should have opportunities to make their own observations based on the context and the graphs. In describing the shape of the data set, students should have opportunities to engage in discussion around data distributions that are symmetric as well as those that are skewed. Students should also be expected to consider clusters, peaks and gaps as seen in dot plots and histograms in their verbal descriptions of data displays. This standard requires students to begin to think statistically, which will be essential as they move on to analyze two numerical data sets in grade 7 (7.SP.B.3).

Level 4:

Students at this level go beyond describing a data set by its center, variability or shape. Students should engage in conversation around each measure of center, measure of variability and shape. They should have opportunities to determine and justify when or why a measure of center would be a better choice for describing a data set. Students should be expected to justify their choice of measure of center using precise mathematical language. Instruction should include discussions around the impact of an extreme value on the mean, median, or mode and the range and challenge the students to explain how the data set might look different if the very low and/or a very high data point(s) were excluded from the data set.

Statistics and Probability (SP)

Standard 6.SP.A.3 Cluster Heading: A. Develop understanding of statistical variability.

Recognize that a measure of center (mean, median, mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a data set on a line plot. Arrange numerical values in order.	Estimate the mean on the graph of a numerical data set. Calculate the mean, median, mode, and range of a given set of numerical data.	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number. Recognize that a measure of variation describes how its values vary with a single number. Distinguish between measures of center and measures of variation. Determine appropriate measures of center and variation for various data sets.	Model with examples the difference between measures of center and measures of spread. Critique the interpretation of data using precise mathematical vocabulary. Explain why two very different sets of data can have the same median and mean but differ by their variability.

Instructional Focus Statements

Level 3:

In grade 5, students created line plots to display data and solve problems related to the information in the line plots. In grade 6, students not only represent data but also analyze the data using measures of center and variation. Through discussion, students should recognize that measures of center summarize data with a single number. Whereas measures of variation use a single number to describe how the values in the data set vary.

Instruction should include opportunities for students to discuss measures of center in data sets and reflect on how each measure can influence the interpretation of the data. In discussing the mean of a data set, students should engage in activities that illustrate the equal distribution of data. When finding the median of data sets, students should be exposed to data sets with an even and odd number of data points. It is essential that students understand that some data sets are bimodal, meaning that two numbers occur more frequently than other data points. It is a common misconception to express the range of data as low to high, such as 8 to 25, so students should be reminded that the range is a single value that describes variation in the data set. The use of contextual examples will allow students to not only understand how to calculate measures but also understand their meaning in terms of the context.

Instruction should include opportunities for students to recognize that measures of center, alone, are not sufficient summaries for statistical data. To illustrate this, instruction should include data sets with the same mean or medians but different variations. In addition, students should have opportunities to not only calculate measures of center and variability, but to also use models to illustrate how the measures reveal different information about a data set.

Level 4:

Students at this level should be challenged to critique an interpretation of data without performing the actual calculations. For example, given a dot plot representing test scores and a peer's statement about the average test score, students should be able to explain whether a peer's interpretation of class performance is valid using precise mathematical vocabulary. Students at this level should also be challenged to critique the use of a single measure when interpreting data involving situations where multiple measures should be considered. Multiple opportunities to create models and engage in discourse about the difference between measures of center and measures of variability will support students to build fluency with justifying the use of measures to interpret data.

Statistics and Probability (SP)

Standard 6.SP.B.4 Cluster Heading: B. Summarize and describe distributions.

Display a single set of numerical data using dot plots (line plots), box plots, pie charts and stem plots.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify various types of graphs used to display numerical data. Represent data using dot plots.	Choose an appropriate scale for a given numerical data set. Calculate the median and range for a given data set. Identify upper quartile, lower quartile, and interquartile range on a box plot and stem plot.	Interpret data that is represented in a dot plot, box plot and stem plot. Interpret data that is represented on a pie chart by relating each section to the whole circle. Create a dot plot, box plot, and stem plot using a numerical data set. Create a histogram using a numerical data set. Create a pie chart using a numerical data set.	Select the most appropriate display to represent given data. Provide context for a given dot plot, box plot, pie chart or stem plot. Explore the relationship between histograms and stem plots.

Instructional Focus Statements

Level 3:

In earlier grades, students constructed pictographs, line plots, and bar graphs and used these representations to solve problems. In standard 6.SP.A.2, students learn to describe numerical data sets using measures of center (mean, median, mode), variation (range only) and overall shape. Students now

expand their knowledge of data representations to include dot plots, box plots, pie charts, stem plots and histograms. Students should understand that data are organized in graphs for the purpose of analyzing the data. Students should gain an understanding of how the values that are represented in the graph and how to describe the shape and distribution of the data. Students should develop procedural fluency by displaying data graphically in a format appropriate for the data set. They should be given opportunities to interpret data from graphs generated by others. Multiple opportunities to practice displaying the same data using different representations will help students develop a conceptual understanding of the benefits of each representation. When constructing graphs, students should be expected to include a title and appropriate scales.

For students, box plots are often the most difficult of the graphical displays because they represent a summary of five data measures (minimum, lower quartile, median, upper quartile, and maximum) and it can be graphed either vertically or horizontally. In order to support students' understanding of this representation, students should engage in discourse around the use and meaning of each measure including the meaning of the box and whiskers in relation to the data points. Ask students to interpret the box plot to give meaning to the context of the data with a summary.

Students should also display data in a histogram. Instruction should include a discussion of the similarities and differences between histograms and bar graphs. A bar graph usually represents a graphical comparison of discrete or categorical data, while a histogram is a representation of the frequency distribution of the variable in a data set. Mathematical discourse should lead students to understand that histograms have very specific properties. Those properties include: the bars of a histogram touch, the vertical axis of a histogram is the frequency

Level 4:

Students at this level should explore contextual sets of data to determine which statistical graph is the most appropriate graph to display the numerical data. Discourse should include opportunities to explain students' choice of graph and why it is most appropriate for the data set. In addition to interpreting data that is presented in one of these graphs, students should also be able to determine what statistical information is not provided in the different types of graphs. For example, a student cannot determine the mean for a data set that is represented on a box plot. Students should be able to generate real-world contextual problems when they are given data that is already displayed in a graph or given a set numerical data.

Students may begin to explore the relationship between box plots and histograms with respect to the data distribution shape or how the intervals on a histogram present different information than the bars on a bar graph.

Statistics and Probability (SP)

Standard 6.SP.B.5 Cluster Heading: B. Summarize and describe distributions.

Summarize numerical data sets in relation to their context.

6.SP.B.5a Report the number of observations.

6.SP.B.5b Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.

6.SP.B.5c Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered.

6.SP.B.5d Relate the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Organize data in numerical order. Identify and define measures of center that are used in statistics. Identify units of measurement for a set of data.	Determine the mean, median, and range for a given set of data that is presented numerically or in a graph. Define statistical terms such as variability and measure of center.	Summarize data displays by describing overall patterns in a distribution. Communicate appropriate measures of center and variability (range) when describing a data set. Analyze data displays to determine the attribute being measured, its unit of measurement and the number of observations. Summarize data using the mean, median and range considering the method used to collect the data.	Construct viable arguments to explain statistical measures and predict how changes in the data affect those statistical measures.

Instructional Focus Statements

Level 3:

In grade 5 (5.MD.B.2), students represented data on line plots and solved problems involving information presented on line plots. In grade 6, students also learn to display data on dot plots, stem plots, and box plots (standard 6.SP.B.4). This standard requires students to summarize data in relation to the context that it is given. For example, students might describe the data by reading the graph's label then use that contextual information to summarize how the data were measured, what units were used, and select an appropriate section of the graph to represent the data. The summary of statistics described can include quantitative measures of center and variability including mean, median, range, quartiles, and interquartile ranges. Students should be presented with stem plots, line plots and other data displays and expected to determine the number of observations or the sample size within the context. When describing data sets, discussion should include measures of center and variability and how extreme values can affect the measure chosen to describe the data. To deepen students' understanding of the effect of extreme values, instruction should include opportunities to explore how adding or removing data points from a data set affects the measures of center and variability. Students might also realize that the mean may not represent the largest cluster of data points and that the median is a more useful measure of center. Additionally, students should be expected to communicate an understanding of overall patterns when analyzing data. Authentic opportunities for students to interpret and summarize data in relation to the context will be essential as they deepen their understanding of statistics in grade 7.

Level 4:

Students learn that data sets can be described and compared using various statistical measures, depending on what characteristics are being identified. At this level of understanding, students understand that the way data is collected, organized, and displayed influences interpretation and can inform decisions. Knowing this, students should be expected to construct viable arguments to explain how misleading data can influence the results. Students should also be expected to use histograms, box plots and other data displays to explain how data can be misleading both intentionally and unintentionally.